ER 116

(NASA-CR-146000) AERODYNAMIC HEATING FOR WEDGE/WEDGE OR CONE/CONE AT ANGLES OF ATTACK FROM ZERO TO APPROXIMATELY 40 DEG: DIGITAL COMPUTER PROGRAM NZLDW005, FORTRAN 4, IBM 360/91 (Fairchild Hiller Corp., Riverdale,

N76-71102

Unclas
00/98 04137

NOTE:

CONE-CYL OPTION IS APPLICABLE
ONLY when $\alpha = 0°$

FOR $\alpha > 0°$ use

TETC1 = fore cone half angle + α (deg.)
TETC2 = (arbitrarily) 0.2 + α (deg.)

and run as a cone/cone problem!

Report No. ER-116

AERODYNAMIC HEATING FOR WEDGE/WEDGE OR CONE/CONE AT ANGLES OF ATTACK FROM ZERO TO APPROXIMATELY 40°: DIGITAL COMPUTER PROGRAM NZLDW005, FORTRAN IV, IBM 360/91

REPORT NO. ER - 116

NOVEMBER 1968

Prepared by:

L. D. Wing

Approved by:

C. G. Thomas
Manager, Flight Performance

E. C. Aaron
Director, Beltsville Operations

Fairchild Hiller Corporation
Technical Services Division

AERODYNAMIC HEATING FOR WEDGE/WEDGE OR CONE/CONE AT ANGLES OF ATTACK FROM ZERO TO APPROXIMATELY 40°: DIGITAL COMPUTER PROGRAM NZLDW005, FORTRAN IV, IBM 360/91

SUMMARY

NZLDW005 is an aerodynamic heating program designed particularly to handle the case of a vehicle composed of a sharp nosed, conical section followed by a conical section of lesser half-angle. The effects of the crossflow arising from low to medium ($<30^{o}$) angle of attack are included. The output consists of heat rate and wall shear stress profiles along the windward generator. A selection may be made by the operator as to whether the pressure on the downstream cone is derived as the asymptotic cone pressure (the pressure which would exist if the entire body consisted of a cone of the after-cone half angle at the applicable angle of attack) or as the pressure resulting from a real gas in equilibruim Prandtl-Meyer expansion from the fore cone to the aft cone. A similar problem consisting of a wedge followed by a wedge of lesser half-angle can be run at the operator's option. The output is analogous to the cone/cone case but there is, of course, no crossflow effect. Whenever a conical body is run, the program calculates and prints out the pressure variation along the downstream cone as derived by the shock-expansion method of Syvertson and Dennis. The pressure distribution so derived is printed out solely as extraneous data and is NOT used in any way in the program heat transfer or friction coefficient calculations. This pressure distribution on the downstream cone is strictly valid only at negligible angle of attack and so should be ignored if the angle of attack is greater than five degrees.

CONTENTS

PURPOSE

The purpose of the IBM digital program NZLDW005 is to compute, for both laminar and turbulent flow, the heat transfer rate from the boundary layer to the wall, the heat transfer coefficient, the local flow conditions external to the boundary layer, the wall shear stress, the friction coefficient, and the boundary layer reference conditions along the windward generator of wedge/wedge or cone/cone bodies at arbitrary (up to $\approx 30^{\circ}$) angle of attack. The program is valid for air only and assumes the flow to be a real gas in equilibrium.

Calculation of the shock-expansion pressure distribution on the downstream cone (only) is included. Though this pressure distribution is NOT used in the program, it is printed out so the operator may estimate the effect of aftcone actual pressures as opposed to the asymptotic cone or Prandtl-Meyer pressure (elective) used by the program. If it appears warranted, the operator can then elect to use the shock-expansion pressure distribution to run additional studies using a heating program capable of accounting for pressure gradients (e.g. NZLDW001, Reference 1).

GENERAL DESCRIPTION

The configurations which an be analysed by NZLDW005 are limited to cone/cone or wedge/wedge bodies having sharp noses or leading edges. In both cases, the downstream cone or wedge half-angle (θ_C or θ_W) must be smaller than the upstream body half-angle. Since the program is provided with a counter which differentiates between two-dimensional and quasi-three-dimensional flow, the term "cone" will be understood to mean "cone or wedge, as applicable" in this section of the report. The configurations are shown in Figure 1 with identifying terminology.

The program requires that the nose shock wave be attached so the bodies investigated must have relatively sharp noses. Obviously, no physically meaningful body has a perfectly sharp nose. Therefore, judgement is required in the application of the program to each individual problem. The degree of nose bluntness will determine the mass of air which will pass through a near-normal shock wave at the nose tip with a consequent sharp increase in the entropy of this air. As the high entropy air passes around the body, it effects the local flow properties external to the boundary layer until it is either absorbed into the boundary layer or swept away from the body by the crossflow arising from the body angle of attack. In general, the most extreme effect of the high entropy air is felt when the angle of attack is zero, but even in this case the effect entends only some five to ten nose diameters down the body from the nose. Since the high entropy jump across the near-normal shock causes a decrease in heat transfer to the body wall from the boundary layer, it is usually conservative to assume an attached (oblique) nose shock. Thus NZLDW005 is quite valid except in the immediate vicinity of the nose. If it is desired to make a detached nose shock analysis to evaluate heating and shear stress conditions around and in the vicinity of the blunt nose, another program (NZLDW001, Reference 1) is available for this purpose.

It is important to bear in mind that if the cone angle (plus the angle of attack) gets large enough to result in separation of the shock wave from the body, this entire analysis is invalid and it will be necessary to switch to a "pure crossflow" type of analysis which can be closely approximated by NZLDW001 (Reference 1). This is the primary reason for limiting the applicability of the program to moderate angles of attack. The "cone angle plus angle of attack" at which the shock would separate from the body varies with the Mach Number but, in general, a reasonably conservative assumption of a maximum value for this angle would be about 40 degrees.

One of the options at the disposal of the operator is the election to use asymptotic pressure or pressure derived from a real gas (in equilibrium) Prandtl-Meyer expansion on the downstream body. Again, judgement is required in making this selection since there is no convenient method for determining which method to use in a specific case. If one considers a body system having an extremely large length to diameter ratio (very slender body), then the pressure somewhere far back on the aft cone or wedge will indeed approach the asymptotic value. However, if the body is relatively stubby, the Prandtl-Meyer expansion pressure is probably a better estimate. Of course, the most accurate method would make use of the shock-expansion pressure distribution on the downstream body (as derived by this program but used as input to a pressure gradient heat transfer analysis program). The present program, it must be remembered, considers the asymptotic pressure (a constant) on the forebody and either the asymptotic or the Prandtl-Meyer (elective) pressure (a constant) on the aft body.

Also worthy of mention is the fact that NZLDW005 automatically calculates each problem considering first a laminar and then a turbulent boundary layer. Both laminar and turbulent values are printed out. It is entirely within the responsibility of the operator to determine which assumption is valid. The machine merely presents both calculations. Clearly, in cases where the turbulent value for a given station is less than the laminar value, then it would be nonsense to assume turbulent flow using the program's analytical techniques - regardless of what any other criteria (such as local Reynolds number or Momentum thickness Reynolds number) might indicate.

In brief summary, the sequence of operations of the program are as follows:

a. Read all input data.

b. Call the atmosphere subroutine: TBLALT (1959 ARDC atmosphere) with the input altitude and come out with all desired "free stream" thermodynamic and transport properties of air.

c. Estimate the pressure and temperature (ambient) behind the nose shock and using the oblique shock real gas routine, converge on the post-shock actual thermal and transport properties of air. These properties are the correct external-to-boundary layer values for the forward wedge (2 dimensional case).

d. Use the energy and momentum conservation equations to proceed from the post-oblique shock conditions to conditions external-to-the-boundary layer on the forward cone surface (axisymmetric case).

e. Obtain asymptotic properties on the aft wedge by the same method as (c), above, but using the downstream body half-angle plus angle of attack.

f. Use the methods of (c) and (d), above, with downstream cone half-angle plus angle of attack to get the downstream cone asymptotic external-to-boundary-layer properties.

g. Use the real gas Prandtl-Meyer expansion routine to obtain the P-M local flow properties on the downstream cone or wedge. (Note that the selection of (f) or (g) for heating and shear stress analysis on the downstream body is a program input).

h. Get wall properties (properties of air at local pressure and wall temperature) on forebody using the asymptotic body pressure.

i. Get boundary layer reference properties on forebody using asymptotic pressure.

j. Get downstream body wall properties using either P-M expansion pressure or asymptotic pressure (as elected in input) on cone or wedge.

k. Get downstream body reference properties using either P-M expansion pressure or asymptotic pressure (as elected in input) on cone or wedge.

l. Get heat rates, friction coefficients and shear stresses on forebody (laminar and turbulent boundary layer).

m. Get laminar heat rates, friction coefficients and shear stresses on aftbody.

n. Get turbulent heat rates, friction coefficients and shear stresses on aftbody. (Note that calculations (m) and (n) will be based upon P-M or asymptotic downstream body pressures depending upon which pressure the operator has requested in input).

o. Get local pressure distribution on downstream cone (only) by Syvertson and Dennis shock expansion method.

The mathematical means of accomplishing the above tasks, (a) through (o), are discussed in the Theory Section and Appendices A, B and C.

DISCUSSION

Certain general observations can be made about NZLDW005 although only repeated comparison with available test data can finally demonstrate the degree of accuracy of the program. The general applicability of the pertinent theory is discussed in the applicable references cited in the "Theory" section of this report. It should be noted, however, that only limited data have been tested against the theoretical prediction of the effects of cross-flow on the heat rates of cones at (up to) moderate angles of attack (note that $\alpha + \theta_C$ should not exceed 40^O and is more probably valid only up to 25 to 30 degrees).

Selection of P-M or Asymptotic Flow on Aftbody -

In general, it will be found that the selection of asymptotic pressure on the aftbody (JJJ = 1) will result in the higher heat transfer rate prediction. It is probable that these heat rates will be too high, particularly for relatively short vehicles (length to base diameter ratios of 10 or less). However, the assumption of Prandtl-Meyer expansion (JJJ = 0) leads to heat rates on the aftbody which are slightly lower than should be anticipated. There is, of course, a distinct danger of overgeneralizing because conditions of flight may produce unexpected variations from the "most probable" results discussed here. When tunnel or test data are available which are representative of the actual flight conditions, it is suggested that data be run both ways, particularly in the regions of maximum heating and the remainder be run using the most successful assumption for the JJJ counter. In the absence of any experimental data, the "best guess" at the theoretical heat rate distribution on the aft cone will probably result from running JJJ = 1 (asymptotic pressure on aftbody) and scaling the resulting aftbody heat rates down by the ratio (at each station) of: (local PLOCC2/PC2) x (QSLAM2) - or QST2 for the turbulent boundary layer.

Since PLOCC2 is the Syvertson and Dennis shock-expansion pressure on the aftbody and is calculated by the program only when cone bodies are run, the above correction is not available for the two dimensional flow case. Thus one can only use the conservative JJJ = 1 value of heat rate for the wedge case or simply run JJJ = 0 and 1 and numerically average the results if less conservatism is desired (this averaging method should yield results which are closer to reality).

Accuracy of Aeroheating Analyses -

The term "realistic" applied to the accuracy of aeroheating analytical methods encompasses a multitude of sins. Although one frequently sees comparisons between analysis and flight or experimental data which "agree within a few percent," the writer is highly skeptical of the general significance of such data. Indeed, such orders of accuracy between theoretical conjecture and apparent measurement do exist. However, the "one to 7.31%" accuracies that one sees quoted can be very misleading. In many practical problems of aeroheating prediction the ability to predict within ten to twenty percent must be considered excellent and a frequent variation of 50 to 100% must be anticipated. There are simply too many approximations and assumptions required in the theories to make them practically applicable over sufficiently wide ranges and measurement methods are similarly restrictive. In general, the lower the total energy of the flow, the lower the discrepancies between conjuration (theory) and observation (experiment).

It must be recognized that many of the theories employed in NZLDW005 have produced documented results within less than 5% but it must also be recognized that these same theories applied in different flight regimes show no such accuracy. It is not the purpose here to criticize available theoretical methods but solely to emphasize the constant need for suspicion on the part of the program user and to implore the greatest possible use of the best experimental evidence to substantiate the program theory in the flight regime in which it is being asked to perform.

Cone-Cylinder Option -

The capability of analysing a sharp cone followed by a cylindrical aftbody <u>AT ZERO</u> angle of attack has been provided. To apply this option, the user sets N = 1 (cone flow) and TETC2 (downstream cone angle plus angle of attack) = 0. The program then sets the downstream flow selector to unity (JJJ = 1) regardless of what the user inputs and the cylinder external-to-boundary-layer properties are the free stream (forward of nose shock) properties for the appropriate altitude and velocity. The effective surface coordinate distance at the fore end of the aft body (S2 MINT and S2 MINL) are calculated and thus the momentum defect across the "expansion" is held constant.

see note on cover sheet

THEORY

This section presents the equations used in NZLDW005 and also gives brief derivations. The applicable expressions are enclosed in boxes for convenience in identifying the exact form of the equations as they are programmed.

Laminar Heat Transfer Rate -

The derivation of the laminar heat transfer rate equation is based upon Eckert's reference enthalpy adaptation of Lee's momentum integral equation for a laminar boundary layer in a pressure gradient.

The initial equation is (Reference 2):

$$N_u^* = \frac{Lh'}{k^*} = 0.35 \left[\frac{\rho_o U_e L}{\mu_o}\right]^{1/2} H_s^* (Pr)^{1/3} \qquad \text{(Eq 1)}$$

where h' is the heat transfer coefficient.

Using the relation

$$\cancel{\dot{q} = \frac{h'}{h_{rec} - h_w}} \qquad h' = \frac{\dot{q}\, C_{P_w}}{h_{rec} - h_w}$$

and solving (Eq 1) for $\dot{q}$ (the heat transfer rate to the wall)

$$\dot{q}_{lam} = \frac{0.35\, k^*}{C_{P_w} L} \left[\frac{\rho_o U_e L}{\mu_o}\right]^{1/2} H_s^* (Pr)^{1/3} (h_{rec} - h_w) \qquad \text{(Eq 2)}$$

but, also from Reference 2,

$$H_s^* = \frac{(\rho_e^*/\rho_o^*)(U_e/U_\infty)(r/L)^N}{\left[(\rho_e^*/\rho_o)(\mu_e^*/\mu_o)(U_e/U_\infty)\int_o^{S/L}\left(\frac{Y}{L}\right)^{2N} d\left(\frac{S}{L}\right)\right]^{1/2}} \qquad \text{(Eq 3)}$$

Noting that $r = s \sin(\theta_c - \alpha)$, $\dot{q}_{lam} = \dot{q}_{lam}(2N+1)^{1/2}$ (Mangler's $\dot{q}_{cone} = \sqrt{3}\ \dot{q}_{2dim}$), and the pressure gradient is zero (hence local flow properties are independent of S/L) get:

$$\dot{q}_{lam} = \frac{0.35\, k^*}{C_{P_W} L} \left[\frac{\rho_o U_\infty . L}{\mu_o}\right]^{1/2} \left[\frac{(2N+1)(\rho_e^*/\rho_o)(U_e/U_\infty)(\mu_o/\mu_e^*)}{S/L}\right]^{1/2} \left(\frac{\mu C_P}{k}\right)^{1/3} \psi \qquad \text{(Eq 4)}$$

where $\psi = h_{rec} - h_w$ and

$$\left.\begin{array}{l} \psi_{Lam} = \left[(1 - \sqrt{Pr^*})\, he + \sqrt{Pr^*} \quad h_o - h_w \right] \\ \psi_{Turb} = \left[(1 - \sqrt[3]{Pr^*})\, he + \sqrt[3]{Pr^*} \quad h_o - h_w \right] \end{array}\right\} \quad \text{(Eq. 5)}$$

and

$$h^* = \text{reference enthalpy} = \frac{he+hw}{2} + 0.22\sqrt{Pr}\,(h_o - h_e) \quad \text{(Eq. 6)}$$

Equation 4 simplifies to the form programmed:

$$\boxed{\dot{q}_{1am} = \frac{0.35k^*}{Cp_W}\,(Pr^*)^{1/3} \left[(2N+1) \quad \frac{U_e \quad \varrho_e^*}{\mu_e^* \quad S} \right]^{\frac{1}{2}} (\psi\, 1am)(QRTL)} \quad \text{(Eq. 7)}$$

When N = 1, Eq. 7 is multiplied by QRTL1 or QRTL2 (Fore or aftbody)

<u>Turbulent Heat Transfer Rate</u> -

The turbulent boundary layer heat transfer rate is calculated by the Flat Plate Reference Enthalpy method which is given in Reference 3 and (subsequently) in Reference 4. Further justification for the use of this method may be found in Reference 5. The full development is recorded in Reference 6 and is based upon the work of E.R.G. Eckert in Reference 7. A brief summary is given here.

From Eckert (Reference 7), the Stanton number for turbulent, incompressible flow on a flat plate is

$$S_t = \frac{C_f}{2} \quad S' = \dot{q} / \varrho_e \; U_e \; (h_{rec} - h_w) \quad \text{(Eq. 8)}$$

According to Eckert (Reference 8), the proportionality factor, S', can (consistently with the reference enthalpy concept) be expressed as

$$S' = \left[Pr^* \right]^{-2/3} \quad \text{(Eq. 9)}$$

Blasius expression for the local skin friction coefficient on a flat plate in incompressible turbulent flow is (pg. 537 of Reference 9):

$$\frac{C_f}{2} = 0.0296 \; (Re_e)^{-0.2} \quad \text{(Eq. 10)}$$

The two dimensional, turbulent, incompressible heat transfer rate can then be obtained from equations 8, 9 and 10 as:

$$\dot{q}_{turb} = 0.0296\,(Re_e)^{-0.2}\,(Pr^*)^{-2/3}\,(\rho_e\ U_e\ g)\,(h_{rec} - h_w) \qquad \text{(Eq. 11)}$$

The compressibility correction according to Hayes and Probstein (Reference 10) is

$$\frac{C_f \text{ compressible}}{C_f \text{ incompressible}} = \left(\frac{\mu_e^*}{\mu_e}\right)^{0.2}\left(\frac{\rho_e^*}{\rho_e}\right)^{0.8} \qquad \text{(Eq. 12)}$$

which is nothing more than the ratio of the compressible to incompressible momentum thickness. Entering this correction (Eq. 12) into (Eq. 11) and accounting for Van Driest's (Reference 11) "factor of ½" to the Reynolds number for conical flow with the selective constant, N, (zero for 2 dimensional, one for conical flow) obtain:

$$\boxed{\dot{q}_{turb} = 0.0954\left[\frac{1+N}{S}\right]^{0.2}\left[\rho_e^*\quad U_e\right]^{0.8}\left[\frac{k_e^*}{Cp_e^*}\right]^{2/3}\left[\frac{\psi_{turb}}{\left(\mu_e^*\right)^{0.466}}\right](QRTT)} \qquad \text{(Eq. 13)}$$

where ψ_{turb} is (Eq. 5, repeated)

$$\boxed{\psi_{turb} = (1 - \sqrt[3]{Pr^*}\,)\,h_e + h_o\,\sqrt[3]{Pr^*} - h_w = (h_{rec} - h_w)_{turb}} \qquad \text{(Eq. 14)}$$

Laminar Friction Coefficient and Shear Stress -

The laminar friction coefficient is developed by the use of Reynold's analogy as follows:

Eq. 8.3.12 (Reference 10) gives

$$C_{f_{lam}} = \frac{2\,\dot{q}\,(Pr^*)^{2/3}}{\rho_e\ U_e\,(h_{rec} - h_w)} \qquad \text{(Eq. 15)}$$

Using Eckert and Tewfik's (Reference 2) $\dot{q}$ value (Eq. 7) combined with (Eq. 15), get

$$C_{f_{lam}} = \frac{2\,(Pr^*)^{2/3}\,(0.35\,k^*)(Pr^*)^{1/3}\,(h_{rec} - h_w)\left[\dfrac{(2N+1)\,U_e\,\rho_e^*}{\mu_e^*\ S}\right]^{1/2}}{\rho_e\ U_e\,(h_{rec} - h_w)\,Cp_W} \qquad \text{(Eq. 16)}$$

Which, with the Blasius constant (0.664) replacing the constant in (Eq. 16) (2 x .35 = .7), becomes

$$\boxed{C_{f_{lam}} = \frac{0.664\ Cp_e^*}{\rho_e\ \ C_{P_W}} \left[\frac{(2N+1)\ (\rho_e^*\ \ \mu_e^*)}{U_e\ \ S}\right]^{\frac{1}{2}} (QRTL)} \qquad \text{(Eq. 17)}$$

The Mangler correction from two dimensional to conical flow ($\sqrt{3}$) is accounted for by letting N = 0 for 2 dimensional and N = 1 for conical flow. The laminar shear stress is given by

$$\boxed{\tau_{Lam} = 0.5\ C_{f_{Lam}}\ \rho_e\ \ U_e^2}$$

Laminar Shear Stress and Friction Coefficient on Cone at Angle of Attack -

Since the program considers cone/cone bodies at angle of attack, it is necessary to correct the conical (though not the wedge) $C_{f_{lam}}$ and τ_{lam} to include the effects of crossflow. This is done as follows:

In accordance with the assumptions defined on page 4 of Reference 12 (also, see these assumptions in Appendix A), the boundary layer momentum equation yields

$$\tau_w\ L = \rho_e\ \ U_e^2\ \left(\theta\ \frac{dL}{ds} + L\ \frac{d\theta}{ds}\right) = \rho_e\ \ U_e^2\ \ \frac{d\ (\theta L)}{ds} \qquad \text{(Eq. 19)}$$

For a flat plate in laminar, incompressible flow

$$\tau_w = 0.332\ \ \rho_e\ \ U_e^2\ \ \sqrt{\frac{\nu_e}{U_e\ \ S}} \qquad \text{(Eq. 20)}$$

and

$$\theta = 2\ \tau_w\ \ S/\rho_e\ \ U_e^2 \qquad \text{(Eq. 21)}$$

NOTE THAT θ = boundary layer momentum thickness
θ_c = cone half-angle

Combining equations 20 and 21 by elimination of S

$$\tau_{w_{inc}} = \frac{0.2204\ U_e\ \ \mu_e}{\theta} \qquad \text{(Eq. 22)}$$

$$\tau_{w_{comp}} \left[\frac{\tau_{w\ inc}}{\tau_{w\ comp}}\right] = 0.2204 \frac{U_e \mu_e}{\theta_{comp}} \cdot \left[\frac{\theta_{comp}}{\theta_{inc}}\right] \qquad \text{(Eq. 23)}$$

substituting (Eq. 23) into (Eq. 19)

$$0.2204\ U_e\ \mu_e\ \frac{L}{\theta} = \rho_e\ U_e^2\ \frac{d\ (\theta L)}{d\ s} \qquad \text{(Eq. 24)}$$

or

$$\frac{0.2204\ \mu_e\ L^2\ ds}{\rho_s\ U_s} = (\theta\ L)\ d\ (\theta\ L)$$

but

$$L = CS\ \frac{\operatorname{Tan}\ (\theta_c + \alpha)}{\operatorname{Tan}\ \theta_c} \qquad \text{(Eq. 25)}$$

so

$$\left[\frac{0.2204\ \mu_e\ C^2}{\rho_e\ U_e}\right]\left[S\right]^{\left[\frac{2\ \operatorname{Tan}\ (\theta_c+\alpha)}{\operatorname{Tan}\ \theta_c}\right]} \left[\frac{1}{1+\frac{2\ \operatorname{Tan}\ (\theta_c+\alpha)}{\operatorname{Tan}\ \theta_c}}\right] =$$

$$= \frac{\theta^2\ C^2}{2} \left(S\right)^{\left[\frac{2\ \operatorname{Tan}\ (\theta_c+\alpha)}{\operatorname{Tan}\ \theta_c}\right]} \qquad \text{(Eq. 26)}$$

also

$$\theta_{\alpha=0} = 0.664 \left[\frac{\mu_e}{\rho_e\ U_e\ S}\right]^{\frac{1}{2}} \frac{S}{\sqrt{3}} \qquad \text{(Eq. 27)}$$

therefore

$$\frac{\theta\ (\alpha = 0)}{\theta(\alpha \neq 0)} = \frac{\sqrt{3}}{\left[1 + \frac{2\ \operatorname{Tan}\ (\theta_c + \alpha)}{\operatorname{Tan}\ \theta_c}\right]^{\frac{1}{2}}} = \sqrt{\frac{3\ \operatorname{Tan}\ \theta_c}{\operatorname{Tan}\ \theta_c + 2\ \operatorname{Tan}\ (\theta_c + \alpha)}} \qquad \text{(Eq. 28)}$$

or, finally,

$$QRATL = \left[\frac{C_{f\,\alpha \neq 0}}{C_{f\,\alpha = 0}}\right]_{Lam} = \left[\frac{\tau_{w\,\alpha \neq 0}}{\tau_{w\,\alpha = 0}}\right]_{Lam} = \left[\frac{1 + \frac{2\,\mathrm{Tan}(\theta_c + \alpha)}{\mathrm{Tan}\,\theta_c}}{3}\right]^{\frac{1}{2}} \qquad \text{(Eq. 29)}$$

The above expression is termed QRTL in the program. From Reynold's analogy, (Eq. 29) is also the correction for the heat transfer rate on a cone windward generator when the cone is at angle of attack since the heat transfer is proportional to the friction coefficient.

Turbulent Friction Coefficient and Shear Stress -

The shear stress equation for a flat plate in turbulent, compressible flow (based on the Blasius solution given on page 536 of Reference 9) is:

$$\tau_{turb} = \frac{(0.0125)\rho_e U_e^2 \left[f(m)\right]^{5/4}}{(U_e/\nu_e)^{\frac{1}{4}}\ \theta^{\frac{1}{4}}} \qquad \text{(2 Dim., Comp.)} \qquad \text{(Eq. 30)}$$

The flat plate momentum thickness is (See Appendix A)

$$\theta = \frac{0.036\ f(m)\ S^{0.8}}{(U_e/\nu_e)^{0.2}} \qquad \text{(2 Dim., Comp.)} \qquad \text{(Eq. 31)}$$

Combining (Eq. 30) and (Eq. 31) and introducing the Van Driest (Reference 11) correction for conical flow (again, in selective form)

$$\tau_{w_{turb}} = 0.0287 \left[\frac{(1+N)\ \mu_e^*}{S}\right]^{0.2} (\rho_e^*)^{0.8}\ (U_e)^{1.8} \quad \text{(ORTT)} \qquad \text{(Eq. 32)}$$

Compressible; two dimensional or axisymmetric where, as before, N = 0 for two dimensional and N = 1 for cone flow

The local friction coefficient is, then,

$$C_{f_{turb}} = \frac{2\ \tau_{w_{turb}}}{\rho_e\ U_e^2} \qquad \text{(Eq. 33)}$$

Turbulent Shear Stress and Friction Coefficient on Cone at Angle of Attack

The development of the turbulent crossflow effect at the windward generator of a cone at angle of attack is taken from unpublished work of Dr. Joseph Sternberg and is presented in detail in Appendix A. Summarizing the results:

$$\left[\frac{\dot{q} \text{ with crossflow}}{\dot{q} \text{ without crossflow}}\right]_{\text{Turb}} = \left[\frac{C_f \text{ with crossflow}}{C_f \text{ without crossflow}}\right]_{\text{Turb}} \quad \text{(Eq. 34)}$$

The development of Appendix A leads to:

$$QRATT = \boxed{\frac{\dot{q} \text{ with crossflow}}{\dot{q} \text{ without crossflow}} = QRTT = 0.85\,(1 + K_2)^{0.2} \quad \text{where} \quad k_2 = 1.25 \left[\frac{\text{Tan}\,(\theta_c + a)}{\text{Tan}\,\theta_c}\right]} \quad \text{(Eq. 35)}$$

Effective Surface Distance on Downstream Body Shape (Lam. and Turb.) -

Since there is a discontinuity in the boundary layer flow at the shoulder between the forward and aft body shapes and since the heat transfer and friction forces on the wall are dependent upon the boundary layer growth, it is necessary to derive an effective body coordinate, S2 MINL or S2MINT, which will account for the flow discontinuity.

1. Turbulent Boundary Layer

For the case of the turbulent boundary layer, the development is based upon the assumption of equal momentum defect across the expansion; that is

$$\rho_1 \, U_1^{\,2} \, \theta_1 = \rho_2 \, U_2^{\,2} \, \theta_2 \quad \text{(Eq. 36)}$$

where the subscript 1 refers to local stream properties upstream of the expansion and 2, downstream of the shoulder. From Reference 9

$$\frac{\theta_1 \, \rho_1 \, U_1^{\,2}}{\delta_1} = H_1 \, \rho_1 \, U_1^{\,2} \text{ and } \frac{\theta_2 \, \rho_2 \, U_2^{\,2}}{\delta_2} = H_2 \, \rho_2 \, U_2^{\,2} \quad \text{(Eq. 37)}$$

which, combined with (Eq. 36), yields

$$\delta_2 = \delta_1 \quad \frac{H_1 \ \varrho_1 \ U_1^2}{H_2 \ \varrho_2 \ U_2^2} \qquad \text{(Eq. 38)}$$

but

$$\delta_{1\,max} = \frac{0.0314\,(\mu_1^*)^{0.2}\,(\varrho_1^*)^{.8}\,(S_{1\,max})^{0.8}\left(\frac{\theta_a \neq 0}{\theta_a = 0}\right)_1}{H_1\,U_1^{0.2}\,\varrho_1} \qquad \text{(Eq. 39)}$$

and also

$$\delta_2 = 0.0314\,(\mu_2^*)^{0.2}\,(\varrho_2^*)^{0.8}\,(S_2)^{0.8}\left(\frac{\theta_a \neq 0}{\theta_a = 0}\right)_2 \qquad \text{(Eq. 40)}$$

or

$$S_2 = \frac{U_2^{0.2}\,H_2\,\varrho_2\,\delta_2\left(\frac{\theta_a \neq o}{\theta_a = 0}\right)_2}{.0314\,(\mu_2^*)^{0.2}\,(\varrho_2^*)^{0.8}} \qquad \text{(Eq. 41)}$$

but substituting δ_2 from (Eq. 38)

$$S_{2_{min_{turb}}} = S_2\ MINT = \left[\frac{(U_2)^{0.2}\,\rho_2\,\delta_{1\,max}\,\frac{H_1\,\varrho_1\,U_1^2}{\rho_2\,U_2^2}\left(\frac{\theta_a \neq o}{\theta_a = 0}\right)_2}{0.0314\,(\mu_2^*)^{0.2}\,(\varrho_2^*)^{0.8}}\right]^{1.25}$$

(Eq. 42)

combining (Eq. 39) with (Eq. 42)

$$S_2\ MINT = \left[\left(\frac{\varrho_1^*}{\varrho_2^*}\right)^{0.8}\left(\frac{U_1}{U_2}\right)^{1.8}\left(\frac{\mu_1^*}{\mu_2^*}\right)^{0.2}\beta\ (S_1)^{0.8}\right]^{1.25} \qquad \text{(Eq. 43)}$$

$$\text{where}\ \beta = \left[\frac{\theta_a \neq o}{\theta_a = 0}\right]_1 \quad \left[\frac{\theta_a \neq o}{\theta_a = o}\right]_2 \qquad \text{(Eq. 44)}$$

$$\beta = \frac{1 + K_{2_1}}{1 + K_{2_2}} \qquad \text{(Eq. 45)}$$

but
$$\left.\begin{aligned} K_{2_1} &= \frac{\operatorname{Tan}(\theta_{c_1} + \alpha)}{\operatorname{Tan}\theta_{c_1}} \\ K_{2_2} &= \frac{\operatorname{Tan}(\theta_{c_2} + \alpha)}{\operatorname{Tan}\theta_{c_2}} \end{aligned}\right\} \qquad \text{(Eq. 46)}$$

So combining (Eq. 46) with (Eq. 44) and (Eq. 45) and simplifying, get

$$\beta = \frac{\operatorname{Tan}\theta_{c_2}\left[\operatorname{Tan}\theta_{c_1} + \operatorname{Tan}(\theta_{c_1} + \alpha)\right]}{\operatorname{Tan}\theta_{c_1}\left[\operatorname{Tan}\theta_{c_2} + \operatorname{Tan}(\theta_{c_2} + \alpha)\right]} \qquad \text{(Eq. 47)}$$

and combining (Eq. 47) with (Eq. 43), one finally obtains

$$\boxed{S_2\ \text{MINT} = \left[\left(\frac{\rho_1^*}{\rho_2^*}\right)^{0.8}\left(\frac{U_1}{U_2}\right)^{1.8}\left(\frac{\mu_1^*}{\mu_2^*}\right)^{0.2}\beta\,(S_{1_{max}})^{0.8}\right]^{1.25}} \qquad \text{(Eq. 48)}$$

Where S2 MINT is the initial value of S for the downstream body, assuming turbulent flow.

2. Laminar Boundary Layer

Starting with the laminar boundary layer thickness equations on both the fore body (sub = 1) and aft body (sub = 2)

$$\delta_{1_{max}} = \rho_1 U_1^2 \left(\frac{0.664\, S_1}{\sqrt{R_{e_{s_1}}}}\right) \frac{C_{P_1}^*}{C_{P_{w_1}}} \left[(1+2N)\frac{\rho_1^*\ \mu_1^*}{\rho_1\ \mu_1}\right]^{\frac{1}{3}} (\dot{q}\ \text{RAT1}) \qquad \text{(Eq. 49)}$$

$$\delta_{2_{min}} = \rho_2 U_2^2 \left(\frac{0.664\, S_2}{\sqrt{R_{e_{s_2}}}}\right) \frac{C_{P_2}^*}{C_{P_{w_2}}} \left[(1+2N)\frac{\rho_2^*\ \mu_2^*}{\rho_2\ \mu_2}\right]^{\frac{1}{3}} (\dot{q}\ \text{RAT2}) \qquad \text{(Eq. 50)}$$

where $\dot{q}\ \text{RAT} = \dfrac{\dot{q}\ \text{with crossflow}}{\dot{q}\ \text{without crossflow}}$

Then, to avoid a discontinuity at the shoulder: $\delta_{1_{max}} = \delta_{2_{min}}$ or, equating the right hand sides of (Eq. 49) and (Eq. 50) and simplifying, one obtains:

$$S_2MINL = \left[\frac{U_1}{U_2}\right]^3 \left[\frac{C_{p_1}^*}{C_{p_2}^*}\right]^2 \left[\frac{C_{P_{W_2}}}{C_{P_{W_1}}}\right]^2 \left[\frac{\rho_1^* \; \mu_1^*}{\rho_2^* \; \mu_2^*}\right](S_1) \left[\frac{\dot{q}\ RAT\ 1}{\dot{q}\ RAT\ 2}\right]^2_{Lam} \qquad \text{(Eq. 51)}$$

Prandtl-Meyer Expansion -

All flow conditions upstream of the expansion are known as is the expansion angle (from $\theta_{c1} - \theta_{c2}$). The program has calculated the velocities on both the fore (V1) and aft (V2) bodies assuming asymptotic pressure values. A velocity step, DELV, is then calculated from

$$DELV = \frac{|V2 - V1|}{12}$$

This DELV is used to calculate that value of $\Delta\theta$ which will yield the DELV value. The program then sums up the $\Delta\theta$ values and compares this sum to $\theta_{c1} - \theta_{c2}$ (the total deflection angle). When $\sum\Delta\theta > (\theta_{c1} - \theta_{c2})$, the machine reverts to the last step and recalculates that value of $\Delta\theta$ using DELV/2. This process is continued until the first reading at which $\sum\Delta\theta$ is again less than $(\theta_{c1} - \theta_{c2})$. The machine method is diagrammed in Figure 4.

Calculation of Shock-Expansion Pressure Distribution on Downstream Cone -

It is of interest to calculate the pressure distribution on the downstream cone such that additional calculations may be made of the heat transfer (in a pressure gradient - using a program such as NZLDW001, Reference 1) if the difference between the pressure distribution and the assumed constant (asymptotic or P-M expansion) pressures is sufficiently large. For this reason, the method of Syvertson and Dennis (Reference 13) is included in this program and the desired pressure distribution on the downstream cone is printed out. The method is briefly outlined here. Note that it is only valid for negligible angle of attack!

The program has already calculated the asymptotic external-to-boundary layer properties on the fore cone and the aft cone, as well as the Prandtl-Meyer expansion properties on the aft cone. For the purpose of this section (only) asymptotic cone properties are subscripted with a "c"

i.e. P_{c1}, P_{c2}, etc.

While the downstream properties derived by Prandtl-Meyer expansion do not have the c subscript.

i.e. P_2, γ_2, M_2, etc.

First, the parameters B_2, Ω_1, Ω_2, and r are obtained from

$$B_2 = \frac{\gamma_2 \; P_2 \; M_2^2}{2\,(M_2^2 - 1)} \qquad \text{(Eq. 52)}$$

$$\Omega_1 = \frac{1}{M_{c_1}} \left[\frac{1 + \frac{(\gamma_{c_1} - 1)}{2} M_{c_1}^2}{\frac{(\gamma_{c_1} + 1)}{2}} \right]^{\frac{\gamma_{c_1} + 1}{2(\gamma_{c_1} - 1)}} \qquad \text{(Eq. 53)}$$

$$\Omega_1 = \frac{1}{M_2} \left[\frac{1 + \frac{(\gamma_2 - 1)}{2} M_2^2}{\frac{(\gamma_2 + 1)}{2}} \right]^{\frac{\gamma_2 + 1}{2(\gamma_2 - 1)}} \qquad \text{(Eq. 54)}$$

$$r = S_1 \; \mathrm{SIN}\, \delta_1' \qquad \text{(Eq. 55)}$$

Then

$$\begin{aligned} \delta_1' &= \text{Fore cone half angle + angle of attack } (\theta_{c_1} + \alpha) \\ \delta_2' &= \text{Aft cone half angle + angle of attack } (\theta_{c_2} + \alpha) \end{aligned} \qquad \text{(Eq. 56)}$$

and the pressure gradient is

$$\left(\frac{\partial P}{\partial S}\right)_2 = \frac{B_2}{r} \left[\frac{\Omega_1}{\Omega_2} \; \mathrm{SIN}\, \delta_1' - \mathrm{SIN}\, \delta_2' \right] \qquad \text{(Eq. 57)}$$

Now, for each station on cone 2 (the downstream cone) solve

$$\boxed{\eta = \left(\frac{\partial P}{\partial S}\right)_2 \left(\frac{S - S_1}{P_{c_2} - P_2}\right)} \qquad \text{(Eq. 58)}$$

Then, the local pressure is defined by

$$\boxed{P_{\text{local on cone 2}} = \mathrm{PL\O CC2} = P_{c_2} - (P_{c_2} - P_2)\, e^{-\eta}} \qquad \text{(Eq. 59)}$$

INPUT

A summary of the input format and definition of input items and units is presented in Figure 2. The input is quite simple, consisting of only two or three card types. For the problem which involves free flight in air, only two card types are required. For any case which considers a cone/cone or wedge/wedge configuration which satisfies the condition that the downstream body angle is less than the upstream body angle but which is proceeded in the flow by some arbitrarily shaped, effectively non-blunt body, a third card type is required. Similarly, the third type input card is included if the problem involves a cone/cone or wedge/wedge body run in a wind tunnel whose fluid medium is air.

The first input card type consists of only one card for each problem and contains two test numbers and the problem title. The first test number (fixed point, in column 2) requires a zero if the body shape is two dimensional (wedge/wedge) or the digit "one" if the flow is conical (cone/cone). If the problem is concerned with a body flying in air (that is, the stream properties preceding the body nose are the actual flight free stream properties), the second test number is entered as zero and no type three cards are used. If, on the other hand, the body satisfies the cone/cone (or wedge/wedge) shape requirements but is either run in a wind tunnel (air only) or has additional body structure upstream of and thus effecting the flow over the cone/cone (wedge/wedge) portion, the second test number (column 6) is entered as the digit "one" and the program data must include the type three card.

The second input card type consists of four cards per problem and contains the flight conditions and the body geometry. The data called out on these cards is entered four items to a card (for the first three cards) and <u>one</u> item on the fourth card; in contiguous fields of 15 (floating point). There are also three fixed point items following the single floating point item on the fourth card. The data so input is shown in Table 1 which summarizes the entire program input.

Input card type three is used only when it is desired to specify a pre-nose shock environment other than that derived from the input altitude and the 1959 ARDC atmosphere (NTUN = 1). This card type consists of only one card per problem which calls the desired "free stream" (ahead of the nose shock) pressure (columns 1-15) and temperature (columns 16-30) (both floating point).

It should be noted that the nose shock wave angle is not a program input but is obtained from the cone data of Mary Romig (Reference 14) for $M \sin(\theta_c + \alpha) \geq 8$ and from Reference 15 for $M \sin(\theta_c + \alpha) < 8$. The data of Romig is estimated (adequately) from the relation.

$$\theta_s = \sin^{-1}\left[\frac{1.0027\, M_\alpha \sin(\theta_c + \alpha) + 0.5567}{M_\alpha}\right]$$

This operation is performed by the program. If the flow is two dimensional, the Bertram and Cook data of Reference 16 is used (the curve fit of these data is given in Appendix B, Eq. B 12).

Note that the counter "LDW" (column 21 of card 5) is normally left blank or given a zero value. This allows the system program interrupts to occur. If it is desired to suppress these program interrupts, a "1" is placed in col. 21, card no. 5.

OUTPUT

The output consists of the following section labeled as shown here:

a. Input Data.
b. Free Stream Properties.
c. External to B. Layer Properties on Fore Cone (or Wedge) (Asymptotic Pressure).
d. External to B. Layer Properties on Aft Cone (or Wedge)(Asymptotic Pressure).
e. External to B. Layer Properties from P.M. Expansion to Down Stream Cone (or Wedge).

f. The Values of QRTL1, QRTL2, QRTT1, QRTT2.
g. Wall Properties on Forebody.
h. Reference Properties on Forebody.
i. Down Stream Body Wall and Reference Properties
j. The Values of S2MINL and S2MINT.
k. Heat and Shear Data on Forebody (includes Local Reynolds No.).
l. Heat and Shear Data on Aftbody (includes Local Reynolds No. and Local Shock Expansion Pressure for Conical Body).

The print-out terminology including dimensions is given in Appendix E, (page 44).

SYMBOLS

a = Speed of sound (ft/sec)

B_2 = Defined in Eq. 52

C_f = Friction Coefficient (-)

C_p = Specific heat at constant pressure (B.t.U/lbm ^oK)

g = Acceleration of gravity = 32.174 (ft/sec^2)

h = Enthalpy (B.t.u./lbm)

H = θ/δ

H_s^* = Defined in Eq. 3

k = Coefficient of thermal conductivity (B.t.U./ft sec ^oK)

K_2 = Defined in Eq.'s 46

L = A character istic body length (ft)

M = Mach number (-)

N = A counter: = 0 for wedge flow; = 1 for cone flow

N_u = Nusselt number (-)

P = Pressure (atmospheres)

PLOCC2 = Shock-expansion pressure on downstream cone (atmosphere)

P_r = Prandtl number (-)

$\dot{q}$ = Heat transfer rate (B.t.u./ft^2 sec)

QRATL = ($\dot{q}$ with crossflow/$\dot{q}$ without crossflow) for laminar boundary layer (-)

QRATT = ($\dot{q}$ with crossflow/$\dot{q}$ without crossflow) for turbulent boundary layer (-)

r = Flow deflection distance (local cone radius) (ft)

R_e = Reynolds number (-)

S = Surface coordinate distance from nose apex (ft)

S' = $(P_\gamma^*)^{2/3}$ (-)

S_t = Stanton number (-)

S2MINL= Effective boundary layer build-up distance just downstream of expansion (assumes a constant momentum defect across the expansion) for laminar boundary layer (ft)

S2MINT= Same as S2MINL but for turbulent boundary layer (ft)

S_o= Surface coordinate distance from nose apex to fore body - nose shoulder (ft)

S_1= Surface coordinate distance from nose apex to forebody - aftbody shoulder (ft)

S_2= Surface coordinate distance from nose apex to aft end of aftbody (ft)

T= Temperature (^oK)

U or V= Velocity (ft/sec)

DELV= V2 - V1

DEL= Fraction by which P and T are stepped in the shock routine (-)

Z= Compressibility Factor (-)

α = Body angle of attack (deg)

β = Defined in Eq. 44

γ = Specific heat ratio (C_p/C_v) (-)

δ = Boundary layer thickness (ft)

δ' = ($\theta_c + \alpha$) (See Eq. 56)

η = Exponent defined in Eq. 58

θ= Boundary layer momentum thickness (ft)

θ_c= Body half-angle (deg)

θ_s= Shock wave angle (deg)

μ = Dynamic viscosity (lbf sec/ft^2)

ν = Kinematic viscosity = μ/ρ (ft^2/sec)

ρ = Density (Slugs/ft^3 = lbf sec^2/ft^4)

τ = Shear stress at wall (lbf/ft^2)

ψ = $h_{rec} - h_w$ (B.t.u/lbm)

Ω_1= Defined in Eq. 53

Ω_2= Defined in Eq. 54

Subscripts:

c=	conical body
comp=	assumes compressible flow
e=	local, external to boundary layer value
inc=	assumes incompressible flow
lam=	considers a laminar boundary layer
rec=	at boundary layer recovery enthalpy (h_{total} X recovery factor) (also see Eq.'s 5)
S=	at local, S, position at or near the body surface
turb=	considers a turbulent boundary layer
w=	at wall temperature and local pressure
o=	evaluated at stagnation or located at stagnation point
1=	refers to upstream cone or wedge
2=	refers to downstream cone or wedge
∞=	at free stream (ahead of nose shock) conditions
$[\alpha = 0]$=	at zero degrees angle of attack
$[\alpha \neq 0]$=	at angle of attack greater than zero degrees

Superscript:

$*$ = Properties evaluated at local reference enthalpy and pressure (see Eq. 6)

REFERENCES

1. "General Aerodynamic Heating"
   Digital Utilization Program FB147
   Wing, L. D.
   Report No. 109 October 1967
   Technical Services Division
   Fairchild Hiller Corporation

2. "Use of Reference Enthalpy in Specifying the Laminar Heat Transfer Distribution Around Blunt Bodies in Dissociated Air"
   Eckert, E. R. G. and Tewfik, O. E.
   Journal of the Aero/Space Sciences
   Vol 27, No. 6, June 1960

3. "An Evaluation of Several Hypersonic Turbulent Heat Transfer Analyses by Comparison With Experimental Data"
   Libby, P. A. and Cresci, R. J.
   WADC TN 57-72 July 1957

4. "An Investigation of Laminar, Transitional and Turbulent Heat Transfer on Blunt Nosed Bodies in Hypersonic Flow"
   Cresci, R. J., MacKenzie, D. A. and Libby, P. A.
   Journal of the Aero/Space Sciences page 401
   Vol 27, No. 6, June 1960

5. "Laminar, Transitional and Turbulent Heat Transfer to a Cone-Cylinder-Flare Body at Mach 8"
   Zakkay, V. and Callahan, C. J.
   PIBAL Report No. 737 February 1962
   AF OSR 2359
   Polytechnic Institute of Brooklyn
   Brooklyn, N. Y.

6. "Heat Transfer Technical Memorandum 23"
   Shaw, T. E., Pergament, H. S. and DiChristian, V.
   M.O.S.D. June 5, 1956
   General Electric Co.
   Philadelphia, Pa.

7. "Survey on Heat Transfer at High Speeds"
   Eckert, E. R. G.
   WADC TR 54-70 April 1954

8. "Engineering Relations for Friction and Heat Transfer to Surfaces in High Velocity Flow"
   Eckert, E. R. G.
   Journal of the Aeronautical Sciences page 585
   Vol 22, No. 8, August 1955

9. "Boundary Layer Theory"
4th. Edition
Schlichting, H.
McGraw Hill Book Co., 2nc. 1960
New York City, N. Y.

10. "Hypersonic Flow Theory"
Hayes, W. D. and Probstein, R. F.
Academic Press 1959
New York City, N. Y.

11. "Turbulent Boundary Layer on a Cone in Supersonic Flow at Zero Angle of Attack"
Van Driest, E. R.
Report No. AL-1042 ZZ March 1951
North American Aviation, Inc.

12. "Aerodynamic Heat Transfer Analysis on an Air Defense Missile Including Fin Heating and Load Data"
Wing, L. D. and Stern, I.
ER 12868 January 1963
Martin, Baltimore

13. "A Second Order Shock Expansion Method Applicable to Bodies of Revolution Near Zero Lift"
Syvertson, C. A. and Dennis, D. H.
NACA Report 1328 1957

14. "Conical Flow Parameters for Air in Dissociation Equilibrium"
Romig, M. F.
Research Report No. 7 May 15, 1960
Convair Scientific Research Lab.
San Diego, California

15. "Equations, Tables and Charts for Compressible Flow"
Ames Research Staff
Report 1135
NACA Ames Aeronatuical Lab., Moffett Field, California

16. "The Correlation of Oblique Shock Parameters for Ratios of Specific Heats from 1 to 5/3 with Application to Real Gas Flows"
Bertram, M. H. and Cook, B. S.
NASA TR R-171 December 1963

17. "Nonlinear Mathmatics"
Saaty, T. L. and Bram, J.
International Series in Pure Mathematics
Eq. 2-37, page 76
McGraw Hill Book Co. 1964
N.Y.C., N.Y.

18. "Approximate Calculation of Hypersonic Conical Flow Parameters for Air in Thermodynamic Equilibrium"
Newman, P. A.
NASA TN D-2058 January 1964
Langley Research Center
Langley Station, Hampton, Va.

19. "Similarity-Rule Estimation Methods for Cones and Parabolic Noses"
Linnell, R. D. and Bailey, J. Z.
Vol 23, No. 8 page 796
Journal of the Aeronautical Sciences August 1956

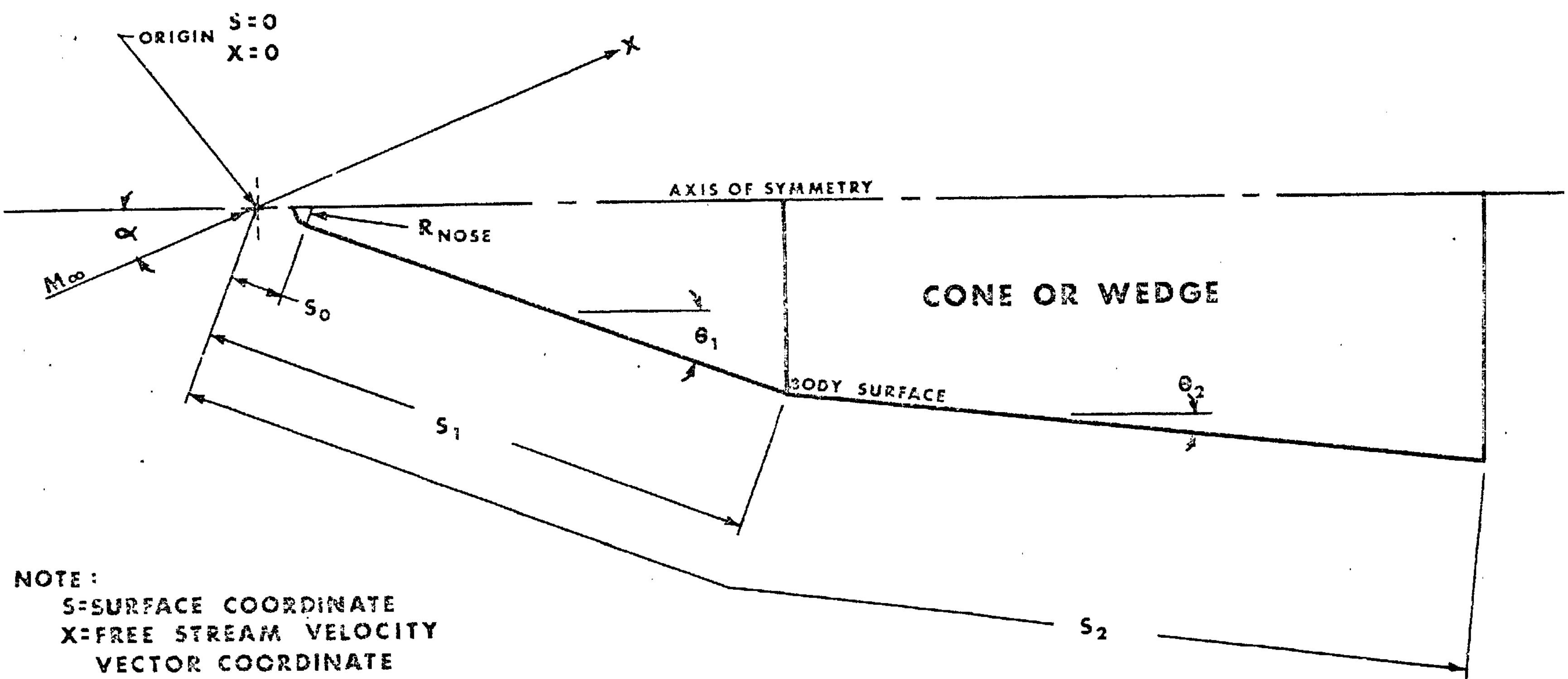
20. "Supersonic Flow Around Right Circular Cones, Tables for Zero Angle of Attack"
Sims, J. L.
Army Ballistic Missile Agency
Report No. DA-TR-11-60
March 1960

21. "Correlation Formulas and Tables of Density and Some Transport Properties of Equilibrium Dissociating Air for Use in Solutions of the Boundary Layer Equations"
Cohen, N. B.
NASA TN D-194 February 1960

TABLE I

INPUT FORMAT FOR NZLDW005

CARD TYPE	CARD NUMBER	ITEM	FORMAT	COLUMNS
1	1	N = 0 (2 dimensional flow); = 1 (conical flow)	Fixed Pt.	2
		NTUN = 0 (ARDC 1959 atmosphere); = 1 (for tunnel case)	Fixed Pt.	4
		Title (any alpha-numeric sequence)	-	7-72
2	1	Vehicle altitude (ft)	Floating Pt.	1-15
		Vehicle velocity (ft/sec)	Floating Pt.	16-30
		DEL = step function for shock routine (use DEL ≈ .01)	Floating Pt.	31-45
		Vehicle angle of attack (deg)	Floating Pt.	46-60
	2	Forebody half-angle plus angle of attack (deg)	Floating Pt.	1-15
		Aftbody half-angle plus angle of attack (deg)	Floating Pt.	16-30
		So (defined in figure 2)	Floating Pt.	31-45
		S1 (defined in figure 2)	Floating Pt.	46-60
	3	S2 (defined in figure 2)	Floating Pt.	1-15
		DEL S1 (defined in figure 2)	Floating Pt.	16-30
		DEL S2 (defined in figure 2)	Floating Pt.	31-45
		Forebody wall temperature (^{o}k)	Floating Pt.	46-60
	4	Aftbody wall temperature (^{o}k)	Floating Pt.	1-15
		LLL = 1 for dump of subroutine "ITER;"= 0 for no dump	Fixed Pt.	17
		JJJ = 0 to call P-M pressure on downstream body = 1 for asymptotic press.	Fixed Pt.	19
		LDW = 0 to allow print of interrupts: = 1 repress interrupts	Fixed Pt.	21
3	1	Free steam pressure (ATM) } (Enter this card only when NTUN - 1)	Floating Pt.	1-15
		Free stream temperature (^{o}k) }	Floating Pt.	16-30



NOTE:
S=SURFACE COORDINATE
X=FREE STREAM VELOCITY VECTOR COORDINATE
O=ORIGIN FOR BOTH COORDINATE SYSTEMS [AT THEORETICAL NOSE APEX]
$\theta_1 > \theta_2$
$S_0 > 0$

FIGURE 1

SKETCH OF GEOMETRY FOR CONE-CONE OR WEDGE-WEDGE PROGRAM FB 172

IBM

FORTRAN Coding Form

X28-7327-5
Printed in U.S.A.

PROGRAM: NZLDW005 - INPUT SUMMARY

PROGRAMMER | DATE | PUNCHING INSTRUCTIONS | GRAPHIC | PUNCH | PAGE OF | CARD ELECTRO NUMBER*

FORTRAN STATEMENT

				Identification Sequence
N NTUN	TITLE			CARD TYPE 1 1
ALT (FT)	VINFY (FT/SEC)	DEL (-)	ALPHA (DEG)	CARD TYPE 2 2
TETC1 (DEG)	TETC2 (DEG)	S 0 (FT)	S 1 (FT)	3
S 2 (FT)	DELTS 1 (FT)	DELTS2 (FT)	TW1 (°K)	4
TW 2 (°K)	LLL JJJ LDW			5
PINFY (ATM)	TINFY (°K)			CARD TYPE 3 6

ITEM	FORMAT
N = 0 for wedge flow; = 1 for cone flow	Fixed point (use col. 2 only)
NTUN = 0 for 1959 ARDC atmos.; = 1 for tunnel cond.'s	Fixed point (use col. 4 only)
Title = Arbitrary alpha-numeric characters	Alpha-Numeric; use col.s 7-72
ALT = vehicle altitude (ft)	Floating point col.s 1-15
Vinfy = vehicle free stream velocity (ft/sec)	Floating point col.s 16-30
DEL = step function for shock routine	Floating point col.s 31-45
Alpha = vehicle angle of attack (deg)	Floating point col.s 46-60
TETC1 = Forebody half-angle + alpha (deg)	Floating point col.s 1-15
TETC2 = Aftbody half-angle + alpha (deg)	Floating point col.s 16-30
SØ = Surf. coordinate dist. from nose apex to forebody - nose shoulder (ft)	Floating point col.s 31-45
S1 = Surf. coordinate dist. from nose apex to forebody - aftbody shoulder (ft)	Floating point col.s 45-60
S2 = Surf. coordinate dist. from nose apex to aft end of aftbody (ft)	Floating point col.s 1-15
Delts1 = Step in S to station forebody (ft)	Floating point col.s 16-30
Delts2 = Step in S to station aftbody (ft)	Floating point col.s 31-45
TW1 = Surf. Temperature on forebody (°K)	Floating point col.s 46-60
TW2 = Surf. Temperature on aftbody (°K)	Floating point col.s 1-15
LLL = 0 (no dump); 1 (dump) iterations in subroutine ITER	Fixed point (right justify in columns 16-17)
JJJ = 0 (P-M press. on aftbody) = 1 (asymptotic press. on aftbody	Fixed point (right justify in columns 18-19)
Pinfy = ambient press. (atm) } enter only if	Floating point col.s 1-15
Tinfy = ambient Temp. (°K) } NTUN = 1	Floating point col.s 16-30
LDW = 0 (program interupts allowed) = 1 to suppress interupts (normally left blank)	

NOTES:

a. Only card type 3 (card no. 6) is optional (used only when NTUN = 1). All other card (1-05) must always be entered.

b. Problems set up as though for individual run, may be stacked in any number of problems.

c. SØ > 0

d. TETC1 > TETC2

*A standard card form, IBM electro 888157, is available for punching statements from this form

FIGURE 2
INPUT SUMMARY

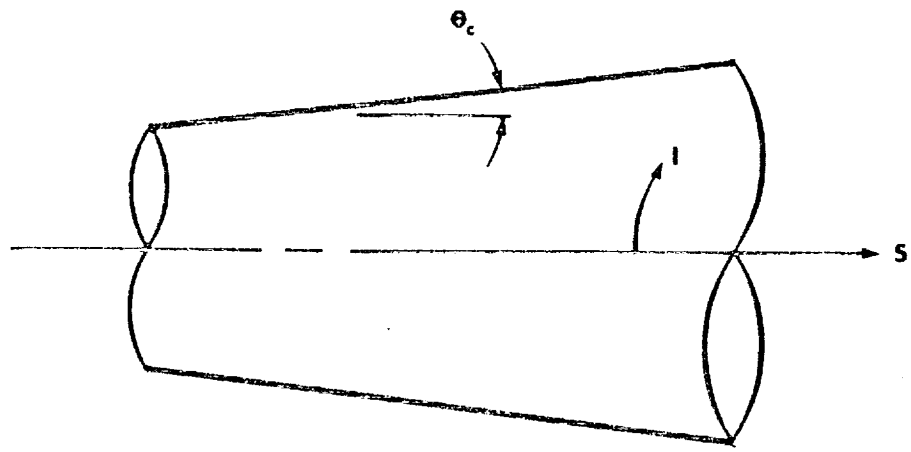


FIGURE 3A

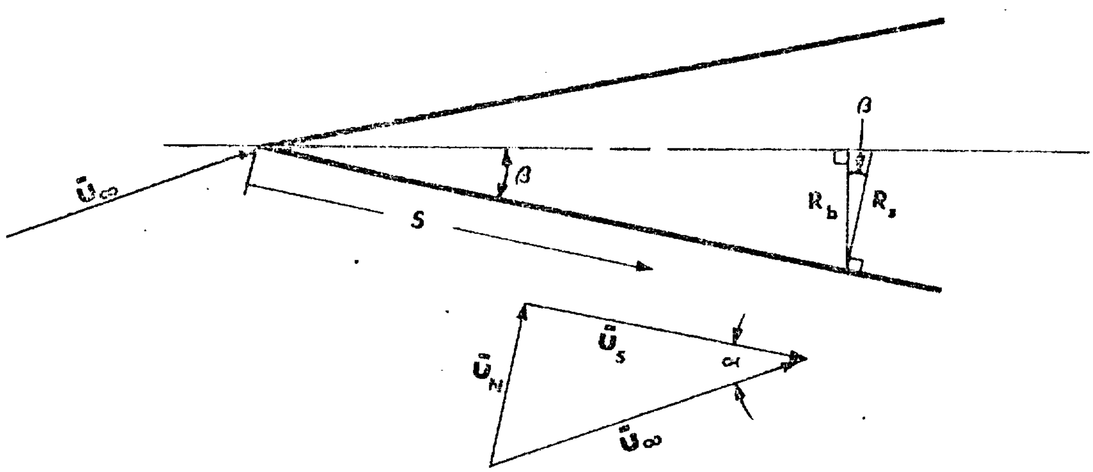
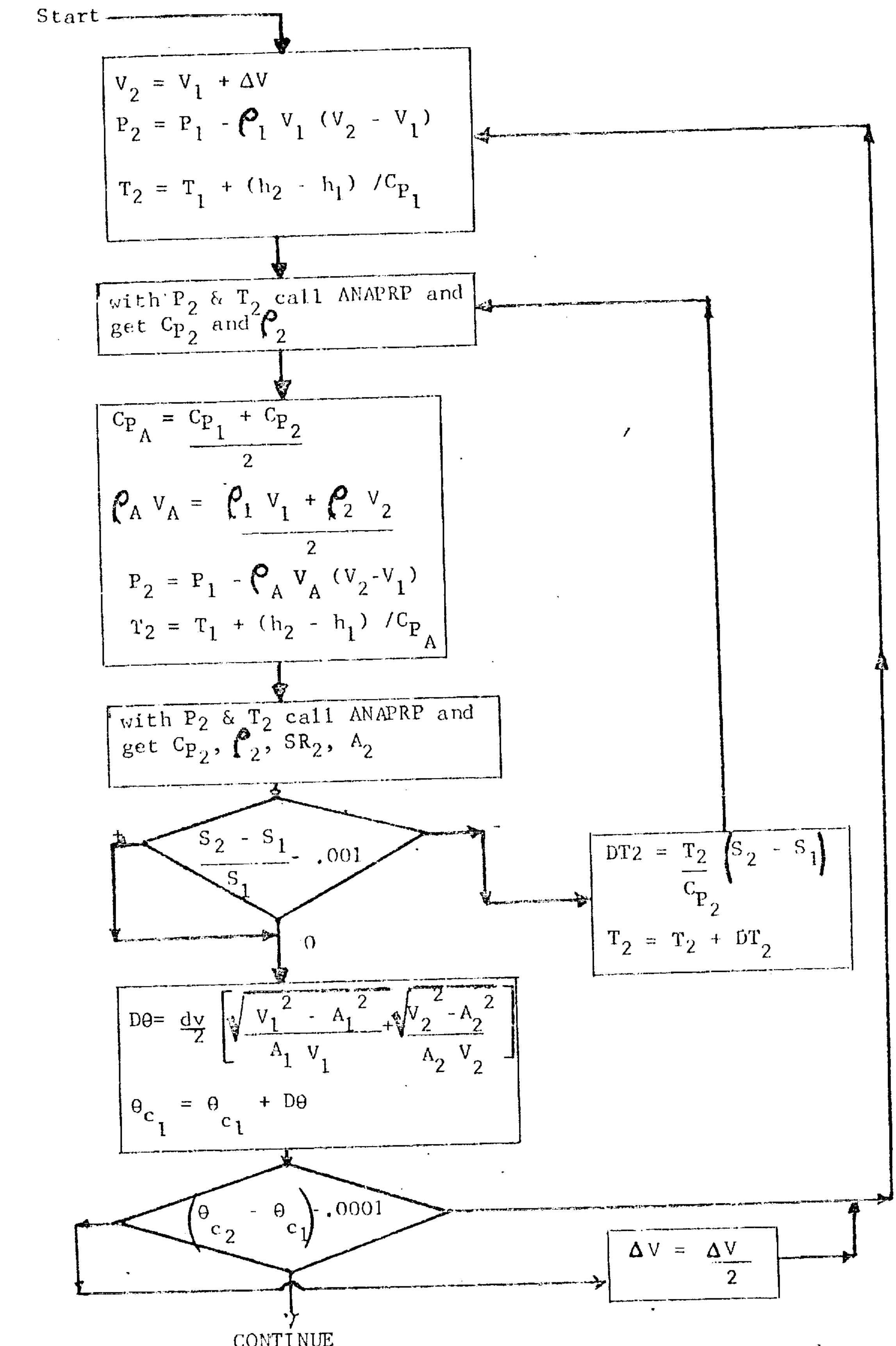


FIGURE 3B

FIGURE 3

SKETCH OF GEOMETRY
(FOR APPENDIX A)

Known: ΔV, θ_{c_2} and all sub 1 values



NOTE: Test (not shown here) causes program to accept last negative value of $\left[\left(\theta_{c_2} - \theta_{c_1}\right) - .001\right]$ after this value passes from negative to positive.

(Sub 2 properties from last entry into ANAPRP are desired values)

FIGURE 4 FLOW DIAGRAM OF REAL GAS PRANDTL-MEYER EXPANSION

EFFECTS OF CROSSFLOW ON CONE HEATING AND SHEAR STRESS

(The following derivation is taken from unpublished work of Dr. Joseph Sternberg). The "bleeding" or "thinning" of the boundary layer arising from the crossflow associated with placing a conical shape at angle of attack causes significant increases in both the heat transfer rate and the wall shear stress. Therefore, the following method is used in the program to account for the crossflow.

According to Reynold's analogy, the heat transfer is proportional to the friction coefficient (C_f) or

$$\frac{\dot{q} \text{ with crossflow}}{\dot{q} \text{ axisymmetric}} = \frac{C_f \text{ with crossflow}}{C_f \text{ axisymmetric}} \qquad \text{(Eq. A1)}$$

where C_f is proportional to the momentum thickness to the minus ¼ power ($C_f \sim \frac{1}{\theta^{.25}}$).

At angle of attack, the boundary layer on the windward side of a cone will be thinned by the crossflow. A simplified theory can be developed for this effect provided the following approximations are valid:

1. The boundary layer thickness is much smaller than the body radius ($\delta \ll R$).

2. The boundary layer velocity distribution can be represented by a 1/7 power law, i.e. $\frac{u}{u_e} = \left[Y/\delta\right]^{1/7}$

3. The spreading of the streamlines in the vicinity of the windward ray can be represented using the Newtonian velocity distribution for the crossflow.

Consider the flow in the vicinity of the windward ray, where (S) is the distance along the windward ray and (ℓ) is the distance normal to S along the surface (figure 3A). The momentum equation for the boundary layer flow is then

$$\tau_w \cdot \ell = \frac{d}{ds}\left[\rho_s \bar{U}_s \theta \ell\right] = \rho_s \bar{U}_s^2 \left[\theta \frac{d\ell}{ds} + \ell \frac{d\theta}{ds}\right] \qquad \text{(Eq. A2)}$$

where:

ρ_s = density outside boundary layer

$\bar{U}_s$ = velocity along S outside boundary layer

$\nu = \mu/\rho$ outside boundary layer

θ = local momentum thickness of boundary layer

τ_w = local friction at the wall

ℓ = a characteristic body surface distance

For a compressible boundary layer, one may write

$$\tau_w = \frac{(0.0125)\ \rho_s\ \bar{U}_s^{\,2}\ \left[f\ (m)\right]^{5/4}}{\left(\bar{U}/\nu\right)^{1/4}\ \theta^{1/4}} \qquad \text{(Eq. A3)}$$

where f (m) is a Mach number correction

$$f\ (m) = \frac{C_f\ \text{comp.}}{C_f\ \text{inc}} \sim \left(\frac{\theta\ \text{inc}}{\theta\ \text{comp}}\right)^{1/4} \qquad \text{(Eq. A4)}$$

combining Eq. A2 with Eq. A3, get

$$\frac{0.0125\ \left[f\ (m)\right]^{5/4}\ \ell}{(\bar{U}/\nu)^{1/4}\ \theta^{1/4}} = \theta\ \frac{d\ \ell}{ds} + \ell\ \frac{d\theta}{ds} \qquad \text{(Eq. A5)}$$

For a cone at zero degrees angle of attack

$$\frac{d\ell}{ds} = 2\pi\ \frac{dr}{ds} \qquad \text{(Eq. A6)}$$

and equation Eq. A5 reduces to

$$\frac{0.0125\ \left[f\ (m)\right]^{5/4}}{(\bar{U}/\nu)^{1/4}\ \theta^{1/4}} = \frac{\theta}{S} + \frac{d\theta}{ds} \qquad \text{(Eq. A7)}$$

The solution to Eq. A7 is in good argrement with the earlier results of Van Driest (reference 11).

In the more general case, the local streamline spreading is given by the streamline angle, $\bar{U}_c/\bar{U}_s$, where $\bar{U}_c$ = the crossflow velocity (in the direction - figure 3B). $\bar{U}_s$ = the flow component along S.

$\bar{U}_c$ is obtained from the crossflow (figure 3B) in which

α = the angle of attack of the ray

β = the cone half angle

R_s = the local body radius

$$R_s = R_b/\cos\beta$$

Assuming the Newtonian approximation for the crossflow

$$\frac{d\,\bar{U}_c}{d\,\psi} = \bar{U}_n/R_s \qquad \text{(Eq. A8)}$$

A more accurate representation of the crossflow would be obtained by using a constant density stagnation point solution. However, the use of the Newtonian velocity gradient results in a streamline divergence angle which is consistent with the cone at zero degrees angle of attack. Then

$$\frac{\bar{U}_c}{\bar{U}_s} = \frac{\bar{U}_n}{R_s} \cdot \frac{\psi}{\bar{U}_s} = \frac{d\psi}{ds} \qquad \text{(Eq. A9)}$$

where

$$\frac{\bar{U}_n}{\bar{U}_s} = \text{Tan}\,\alpha \text{ and } R_b = S \text{ SIN } \beta$$

Thence

$$\frac{0.0125\,\left[f\,(m)\right]^{5/4}}{(\bar{U}/\nu)^{1/4}\ \theta^{1/4}} = \frac{\theta}{S}\ \frac{\text{Tan}\,\alpha}{\text{Tan}\,\beta} + \frac{d\,\theta}{d\,s} \qquad \text{(Eq. A10)}$$

Now introducint a new variable

$$V = \theta^{5/4} \qquad \text{(Eq. A11)}$$

and letting

$$\Lambda = \frac{0.0125\,\left[f\,(m)\right]^{5/4}}{(\bar{U}/\nu)^{1/4}} \qquad \text{(Eq. A12)}$$

Eq A10 transforms to

$$\frac{dv}{ds} + \frac{5}{4}\ V\left[\frac{\text{Tan}\,\alpha}{S\ \text{Tan}\,\beta}\right] - \frac{5}{4}\ \Lambda = 0 \qquad \text{(Eq. A13)}$$

The solution of Eq. A13 is

$$V = C\,S^{-k_2} + \frac{5}{4}\ \frac{\Lambda S}{\left[1 + K_2\right]} \qquad \text{(Eq. A14)}$$

where

$$K_2 = \frac{5}{4}\ \frac{\text{Tan}\,\alpha}{\text{Tan}\,\beta}$$

Then considering that $V = 0$ for $S = 0$, $C = 0$ and there results

$$\theta = \frac{0.0359\ f\ (m)}{(\bar{U}/\nu)^{1/5}} \qquad \frac{S^{4/5}}{[1 + K_2]^{4/5}} \qquad \text{(Eq. A15)}$$

For a cone at zero degrees angle of attack, $K_2 = \frac{5}{4}$ and

$$\theta = \frac{0.036\ f\ (m)\ S^{4/5}}{(\bar{U}/\nu)^{1/5}} \quad \text{or} \quad \frac{\theta\ \text{cone}}{\theta\ \text{plate}} = 0.52 \qquad \text{(Eq.s A16)}$$

which agrees well with Van Driest's result.

Then, for the same distance, S, and the same local flow on the windward meridian, the thinning of the boundary layer on a cone is given by

$$\frac{\theta_\alpha}{\theta_{\alpha = 0}} = \frac{\theta_\alpha}{\theta_c} = \frac{1.91}{\left[1 + K_2\right]^{4/5}} \qquad \text{(Eq. A17)}$$

where

$$\alpha = \beta \text{ and } \frac{\theta\ \alpha}{\theta_{\alpha = 0}} = 1$$

Therefore

$$\boxed{\frac{\dot{q}\ \text{with crossflow}}{\dot{q}\ \text{without crossflow}} = 0.85\ (1 + K_2)^{0.2} \qquad \text{where} \quad K_2 = 1.25 \left[\frac{\text{Tan}\ (\theta_c + \alpha)}{\text{Tan}\ \theta_c}\right]}$$

(Eq. A18)

(Eq. A18A)

APPENDIX B

THE REAL GAS OBLIQUE SHOCK ROUTINE (HYPERSONIC)

This appendix presents the real gas (in equilibrium) oblique shock routine as it is programmed. First the theoretical derivation is given and then the method used to force convergence.

DERIVATION

The oblique shock relation (Eq. 7.12.12, page 269 of Reference 10)

$$P_2 - P_1 = \varrho_1 \; V_1^{\;2} \; \mathrm{SIN}^{\;2} \; \theta_s \left(1 - \frac{\varrho_1}{\varrho_2}\right) \qquad \text{(Eq. B1)}$$

is divided by P_1 and the relation

$$\frac{\varrho_1 \; V_1^{\;2}}{P_1} = \gamma_1 \; M_1^{\;2} \; \mathrm{SIN}^2 \; \theta_s \qquad \text{(Eq. B2)}$$

is introduced to yield the following form of the momentum equation:

$$\frac{P_2}{P_1} = \left(1 - \frac{\varrho_1}{\varrho_2}\right) \; \gamma_1 \; M_1^{\;2} \; \mathrm{SIN}^2 \; \theta_s + 1 \qquad \text{(Eq. B3)}$$

From the Hugoniot relation (see, for example, Eq. 1.4.7, page 12 of Reference 10)

$$h_2 - h_1 = \frac{P_2 - P_1}{2 \; \varrho_1} \left(1 + \frac{\varrho_1}{\varrho_2}\right) = \tfrac{1}{2} \left(P_2 - P_1\right) \left(\frac{1}{\varrho_1} + \frac{1}{\varrho_2}\right) \qquad \text{(Eq. B4)}$$

combined with (Eq. 1.4.5b, page 12, Reference 10)

$$P_2 - P_1 = \varrho_1 \; V_1^{\;2} \left(1 - \frac{\varrho_1}{\varrho_2}\right) \qquad \text{(Eq. B5)}$$

to get

$$h_2 - h_1 = \frac{V_n^{\;2}}{2} \left[1 - \frac{\varrho_1}{\varrho_2}\right] \left[1 + \frac{\varrho_1}{\varrho_2}\right] \qquad \text{(Eq. B6)}$$

dividing Eq. B6 by h_1

$$\frac{h_2}{h_1} = 1 + \frac{V_n^{\;2}}{2} \left[1 - \left(\frac{\varrho_1}{\varrho_2}\right)^2\right] \qquad \text{(Eq. B7)}$$

but

$$V_n^{\;2} = \left(\gamma_1 - 1\right) M_1^{\;2} \; \mathrm{SIN}^{\;2} \; \theta_s \qquad \text{(Eq. B8)}$$

so

$$\frac{h_2}{h_1} = 1 + \left(\frac{\gamma_1 - 1}{2}\right) M_1^{\,2} \text{ SIN}^{\,2}\,\theta_s + 1 \quad \text{[ENERGY]} \qquad \text{(Eq. B9)}$$

Since momentum and energy (normal components) are conserved across the shock, one can write Eq.s B3 and B9 in the form

$$M = P_1 \left[\left(1 - \frac{\varrho_1}{\varrho_2}\right) \gamma_1 \; M_1^{\,2} \text{ SIN}^{\,2}\,\theta_s + 1 \right] - P_2 . \qquad \text{(Eq. B10)}$$

and

$$E = h_1 \left[\left\{ 1 - \left(\frac{\varrho_1}{\varrho_2}\right)^2 \right\} \left(\frac{\gamma_1 - 1}{2}\right) M_1^{\,2} \text{ SIN}^{\,2}\,\theta_s + 1 \right] - h_2 \qquad \text{(Eq. B11)}$$

The method consists of assuming a pressure and temperature behind the shock (viz. a shock angle, θ_s) and solving Eq.s B10 and B11 to obtain M and $E \approx 0$. This is, of course, an interative proceedure.

FORCED CONVERGENCE METHOD

The following convergence method is used to determine the oblique shock angle - hence downstream properties:

1. An original estimate is made of θ_s from curve fit equations for the shock angle in two dimensional or conical body cases.

See pg. 39A

(Wedge flow):

$$\cancel{\text{SIN } \theta_s = \frac{.4922}{M_1} \left[(\gamma_1 + 1) \frac{M_1^{\,2} \text{ SIN } \theta_{wedge}}{\sqrt{M_1^{\,2} - 1}} \right] + \frac{.3889}{M_1}} \qquad \text{(Eq. B12)}$$

(cone flow)

$$\text{SIN } \theta_s = 1.0027 \text{ SIN } \theta_{cone} + \frac{.5567}{M_1} \qquad \text{(Eq. B13)}$$

2. Using the applicable θ_s from Eq.s B12 or B13, values for PTWO and TTWO (downstream of shock pressure and temperature) are estimated using

$$P_2 = P_1 \left((2\,\gamma_1\, M_1^{\,2} \text{ SIN}^2\,\theta_s - (\gamma_1 - 1)) \,/\, (\gamma_1 + 1) \right) \qquad \text{(Eq. B14)}$$

$$V_t = V_1 \cos \theta_s \qquad \text{(Eq. B15)}$$

$$H_2 = \text{HTOT} - (V_t^{\,2} / 50123.) \qquad \text{(Eq. B16)}$$

and then,

$$T_2 = H_2 \,/\, .432 \qquad \text{(Eq. B17)}$$

These values of P_2 and T_2 are the initial estimates, with which one enters ANAPRP (Mollier Subroutine) and obtains ρ_2 and (thence) ρ_1/ρ_2.

3. The momentum and energy equations for the normal component of the velocity are then solved for M and E. (Note that SUB 1 = free stream and SUB 2 = downstream of shock)

$$M = P_1 \left[(1 - \rho_1/\rho_2)\ \gamma_1\ M_1^2 \ SIN^2 \theta_s + 1 \right] - P_2 \qquad \text{(Eq. B18)}$$

$$E = h_1 \left[\left\{ 1 - (\rho_1/\rho_2)^2 \right\} \left(\frac{\gamma_1 - 1}{2}\right) M_1^2 \ SIN^2 \theta_s + 1 \right] - h_2 \qquad \text{(Eq. B19)}$$

In order to conserve momentum and energy (normal components) across the shock one seeks M' = E' = 0 or some values acceptably near to this identity. Thus define

$$F = (M')^2 + (E')^2 \qquad \text{(Eq. B20)}$$

and if $F\text{-}10 \leq 0$, the values of M' and E' are acceptably near to zero and the post shock values are obtained from the values of P_2 and T_2 from Eq.s B14 and B17 and the value of θ_s from Eq. B12 or B13 is the correct shock angle.

4. If, however, $F > 10$, one seeks to alter P_2 and T_2 in such a manner as to converge F to a value less than 10. To accomplish this, first obtain a mathematical statement for the rate of change of M and E by differentiating Eq.s B7 and B8 to get the following four relations:

$$\left(\frac{\partial M}{\partial P_2}\right) = P_1 \left[\gamma_1\ M_1^2 \ SIN^2 \theta_s \left(\rho_1 / \rho_2^2\right) \left(\frac{\partial \rho_2}{\partial P_2}\right)_{T_2} \right] - 1 \qquad \text{(Eq. B21)}$$

$$\left(\frac{\partial M}{\partial T_2}\right)_{P_2} = P_1 \left[\gamma_1\ M_1^2 \ SIN^2 \theta_s \left(\frac{\rho_1}{\rho_2^2}\right) \left(\frac{\partial \rho_2}{\partial T_2}\right)_{P_2} \right] \qquad \text{(Eq. B22)}$$

$$\left(\frac{\partial E}{\partial P_2}\right)_{T_2} = h_1 \left[\left(\frac{\gamma_1 - 1}{2}\right) M_1^2 \ SIN^2 \theta_s \left(\frac{2\rho_1^2}{\rho_2^3}\right) \left(\frac{\partial \rho_2}{\partial P_2}\right)_{T_2} \right] - \left(\frac{\partial h_2}{\partial P_2}\right)_{T_2} \qquad \text{(Eq. B23)}$$

$$\left(\frac{\partial E}{\partial T_2}\right)_{P_2} = h_1 \left[\left(\frac{\gamma_1 - 1}{2}\right) M_1^2 \ SIN^2 \theta_s \left(\frac{2\rho_1^2}{\rho_2^3}\right) \left(\frac{\partial \rho_2}{\partial T_2}\right)_{P_2} \right] - \left(\frac{\partial h_2}{\partial T_2}\right)_{P_2} \qquad \text{(Eq. B24)}$$

It is necessary to use multiple entries into the Mollier data (ANAPRP) in order to define all partial derivatives on the right hand sides of Eq.s B 21, B 22, B 23 and B 24.

Thence define

$$\left.\begin{aligned} P2A &= PTW\emptyset + DEL\ (PTW\emptyset) \\ P2B &= PTW\emptyset - DEL\ (PTW\emptyset) \\ T2A &= TTWO + DEL\ (TTW\emptyset) \\ T2B &= TTWO - DEL\ (TTWO) \end{aligned}\right\} \qquad \text{Eq.s (B25)}$$

Now with PTWO and T2A enter ANAPRP and get RHØ21, H21
with PTWO and T2B enter ANAPRP and get RHØ22, H22
with TTWO and P2A enter ANAPRP and get RHØ23, H23
with TTWO and P2B enter ANAPRP and get RHØ24, H24

One can now define the desired partials as

$$\left(\frac{\partial H_2}{\partial P_2}\right)_{T_2} = \frac{H23 - H24}{P2A - P2B} \qquad \text{(Eq. B26)}$$

$$\left(\frac{\partial H_2}{\partial T_2}\right)_{P_2} = \frac{H21 - H22}{T2A - T2B} \qquad \text{(Eq. B27)}$$

$$\left(\frac{\partial \rho_2}{\partial P_2}\right)_{T_2} = \frac{RH\emptyset 23 - RH\emptyset 24}{P2A - P2B} \qquad \text{(Eq. B28)}$$

$$\left(\frac{\partial \rho_2}{\partial T_2}\right)_{P_2} = \frac{RH\emptyset 21 - RH\emptyset 22}{T2A - T2B} \qquad \text{(Eq. B29)}$$

Equations B26, B27, B28 and B29 are used to solve equations B21 thru B24. One now defines the change of energy and momentum as:

$$dM = \left(\frac{\partial M}{\partial P_2}\right)_{T_2} \Delta P_2 + \left(\frac{\partial M}{\partial T_2}\right)_{P_2} \Delta t_2 = M' \qquad \text{(Eq. B30)}$$

and

$$dE = \left(\frac{\partial E}{\partial P_2}\right)_{T_2} \cdot \Delta P_2 + \left(\frac{\partial E}{\partial T_2}\right)_{P_2} \Delta T_2 = E' \qquad \text{(Eq. B31)}$$

Solving equations 30 and 31 simultaneously for ΔP and ΔT, get

$$\Delta P_2 = \frac{M \left(\frac{\partial E}{\partial T_2}\right)_{P_2} - E \left(\frac{\partial M}{\partial T_2}\right)_{P_2}}{\left(\frac{\partial E}{\partial P_2}\right)_{T_2} \left(\frac{\partial M}{\partial T_2}\right)_{P_2} - \left(\frac{\partial E}{\partial T_2}\right)_{P_2} \left(\frac{\partial M}{\partial P_2}\right)_{T_2}} \qquad \text{(Eq. B32)}$$

and

$$\Delta \quad T_2 = \frac{M\left(\frac{\partial E}{\partial P_2}\right)_{T_2} - E\left(\frac{\partial M}{\partial P_2}\right)_{T_2}}{\left(\frac{\partial M}{\partial P_2}\right)_{T_2}\left(\frac{\partial E}{\partial T_2}\right)_{P_2} - \left(\frac{\partial E}{\partial P_2}\right)_{T_2}\left(\frac{\partial M}{\partial T_2}\right)_{P_2}} \qquad \text{(Eq. 33)}$$

Then P_2 and T_2 are redefined (using Eq.'s B32 and B33)

$$P_2 = P_2 + \Delta P_2$$
$$T_2 = T_2 + \Delta T_2 \qquad \text{(Eq. B34)}$$

With the new values of P_2 and T_2 from equations B34, one resolves Eq. B20 and checks to see that $F = (M')^2 + (E')^2 < 10$. The interation is continued until this inequality is satisfied. At this point all conditions relative to a two dimensional oblique shock are known - hence the external-to-boundary-layer properties on the wedge are known.

It remains only to account for the conical flow field for cases in which N = 1 (cone case). Accordingly, define

$$\text{Tan } \theta_{c_i} = \frac{\left[2 - \rho_1/\rho_2\right] \tan \theta_s}{2 + (\rho_1/\rho_2) \tan^2 \theta_s} \qquad \text{(Eq. B35)}$$

Using the values of θ_s and ρ_1/ρ_2 derived from the last pass through the preceeding routine, calculate the θ_{c_i} value from Eq. B35. Then define

$$d\,\theta_s = \theta_c - \theta_{c_i} \qquad \text{(Eq. B36)}$$

check to see if $\left|\frac{d\,\theta_s}{\theta_c}\right| < .002$. If not, define

$$\theta_s = \theta_s + d\theta_s \qquad \text{(Eq. B37)}$$

and return to the oblique shock routine. When the inequality is finally satisfied, define the cone surface pressure as

$$P_c = P_2 + \rho_1 \; V_1^2 \; \frac{\rho_1}{\rho_2} \; \text{SIN}^2 \; \theta_s \qquad \text{(Eq. B38)}$$

With this pressure (P_c) and T_2 enter the Mollier subroutine (ANAPRP) and get SRC (the entropy at P_c and T_c estimated by T_2). Now, since the post shock (sub 2) and external-to-boundary-layer (sub c) entropies are equal, one seeks to satisfy the inequality:

$$\left| \frac{SR2 - SRC}{SR2} \right| < .0001 \quad \text{(Eq. B39)}$$

When this inequality is satisfied, the cone surface pressure (Pc) and temperature (Tc) are known and thence (through the Mollier data) all local cone surface properties. If equation B39 is not satisfied define

$$DSR = SR2 - SRC$$
$$DTC = (Tc/CPC) \times DSR$$
$$TC = TC + DTC$$

and go back through the Mollier data, repeating the process until Eq. 39 is satisfied.

NOTE

Using these expressions (wedges) and the Mary Romig expression (cones), the resulting θ_S is assumed to be close enough to the real gas value to be acceptible. The momentum and energy equations are then used to obtain the downstream properties (real gas in equilibrium) that are compatible with θ shock. Thus, P_2 and T_2 depend upon iteration across the shock but θ_S is not obtained by iteration but rather from the appropriate curve-fit of real gas equilibrium data.

PERFECT GAS WEDGE SHOCK ANALYSIS FOR N26DW005

(to be used when Real gas wedge shock calculation fails)

1.) Get θ_s from present "estimate" method.

2.) Get p_2 from

$$p_2 = \left(\frac{2\gamma_1 M_1^2 \sin^2\theta_s - \gamma_1 + 1}{\gamma_1 + 1} \right) p_1$$

3.) Get ρ_2 from

$$\rho_2 = \left(\frac{(\gamma_1+1) M_1^2 \sin^2\theta_s}{(\gamma_1-1) M_1^2 \sin^2\theta_s + 2} \right) \rho_1$$

4.) Get T_2 from

$$T_2 = \left(\frac{\rho_1 \, p_2}{\rho_2 \, p_1} \right) T_1$$

These equations check Table II of NACA TR 1135 exactly!

APPENDIX C

SUPERSONIC CONE DATA CALCULATION

The hypersonic similarity assumptions used in the cone shock analysis of Appendix B are invalid when M_∞SIN θ_s is less than approximately 3. Thus, another method is automatically substituted for obtaining the cone shock and surface (external to the boundary layer) properties. This method is essentially a perfect gas analysis and makes use of the curve fitting of Kopal's data by Linnell and Bailey (Reference 19). In order to simplify the approach (and also to assure that the hypersonic analysis will not be attempted when M_∞ SIN $\theta_s < 3$), the program tests the parameter: M_∞SIN θ_c. When the value of this parameter (which is always smaller than M_∞ SIN θ_s for a given cone half angle and M_∞) is equal to or less than 3, the following alternate approach is used.

The pressure at the cone surface is obtained from Linnell and Bailey's (Reference 19) equation

$$C_p = 4\ \mathrm{SIN}^2\ \theta_c \left[\frac{2.5 + 8\sqrt{M_\infty^2 - 1}\quad \mathrm{SIN}\ \theta_c}{1 + 16\sqrt{M_\infty^2 - 1}\quad \mathrm{SIN}\ \theta_c}\right] \qquad \text{(Eq. C1)}$$

where

$$C_p = \frac{P_c - P_\infty}{\frac{1}{2}\rho_\infty V_\infty^2} \qquad \text{(Eq. C2)}$$

or

$$P_c = C_p \left[\frac{(\rho_\infty/2)\ (V_\infty^2)}{2116.2}\right] + P_\infty \qquad \text{(Eq. C3)}$$

Note that in this Appendix (C), C_p is the pressure coefficient, sub ∞ refers to free stream values ahead of the nose shock and sub c refers to properties taken external to the boundary layer at the cone surface.

Data for the local enthalpy at the cone surface is taken from a curve-fit of Figure 14 of Reference 20. The equation is

$$\mathrm{Ln}\left(\frac{h_c}{h_\infty}\right) = 0.40835 \left[M_\infty\ \mathrm{SIN}\ \theta_c\right] - 0.08167 \qquad \text{(Eq. C4)}$$

from which the local enthalpy is defined by

$$h_c = \left\{\mathrm{Ln}^{-1}\left[\ln\ \left(\frac{h_c}{h_\infty}\right)\right]\right\} h_\infty \qquad \text{(Eq. C5)}$$

The local velocity, V_c, is then obtained from the energy equation

$$V_c = \sqrt{(h_{tot} - h_c)\ 50123.} \qquad \text{(Eq. C6)}$$

The local density is estimated from (Reference 21)

$$\rho_{c\ (estimated)} = \left[\frac{0.0294\ (P_c)^{.965}}{1. - 1.0477\ (1 - \frac{h_c}{8465})} \right] \qquad \text{(Eq. C7)}$$

and the temperature (also estimated since it is based upon an estimated density) from

$$T_{est} = 0.684 \left(P_c / \rho_{c_{estimated}} \right) \qquad \text{(Eq. C8)}$$

With P_c and h_c known from equations C3 and C5, respectively, and the estimated temperature, T_{est}, the subroutine ITER is used to correct the estimated temperature to conform to the two known thermodynamic variables (h_c and P_c) using the Mollier data subroutine ANAPRP. This latter subroutine then supplies all of the remaining external to the boundary layer flow properties on the cone.

It remains only to specify the cone shock angle which is obtained from

$$\theta_s = SIN^{-1} \left[\frac{1. - COS\ \theta_c + \sqrt{1. + .5\ (\gamma_\infty + 1)\ (M_\infty^2 - 1)\ (SIN^2\ \theta_c)}}{M_\infty} \right] \qquad \text{(Eq. C9)}$$

(This is Equation 9 of Reference 14)

PERFECT GAS PRANDTL-MEYER EXPANSION

At low velocities the real gas Prandtl-Meyer expansion routine becomes inapplicable and indeed blows up. Thus, when failure of this routine is encountered, the program switches to the perfect gas routine described below.

THEORY

The pressure downstream of the expansion is determined from equation 174 of Reference 15:

$$\frac{P_2}{P_1} = 1 - \frac{\gamma_1 M_1^2}{\sqrt{M_1^2 - 1}} (\Delta \nu) + \gamma_1 M_1^2 \frac{(\gamma_1 + 1) M_1^4 - 4 (M_1^2 - 1)}{4 (M_1^2 - 1)^2} (\Delta \nu)^2$$

$$- \frac{\gamma_1 M_1^2}{2 (M_1^2 - 1)^{3.5}} \left[\frac{\gamma_1 + 1}{6} M_1^8 - \frac{5 + 7\gamma_1 - 2\gamma_1^2}{6} M_1^6 + \frac{5}{3} (\gamma_1 + 1) M_1^4 - 2 M_1^2 + \frac{4}{3} \right] (\Delta \nu)^3 \qquad \text{(Eq. D1)}$$

where $\Delta \nu$ (delta Nu) = the change in Prandtl-Meyer angle through the expansion

sub 1 = upstream of expansion value
sub 2 = downstream of expansion value
γ = specific heat ratio
M = mach number

It should be noted that $\Delta \nu$ is known from

$$\Delta \nu = TETC1 - TETC2 \qquad \text{(Eq. D2)}$$

The program solves Eq.s D1 and D2 for P2. Since the expansion is isentropic, the upstream body entropy is known. Thus, the downstream temperature is approximated by using the asymptotic downstream body temperature and these values (T2 and P2) are used to enter the Mollier data subroutine (ANAPRP) to get the downstream entropy (SR2). This entropy (SR2) is then checked against the known upstream entropy (SR1) and if sufficiently accurate comparison fails the temperature estimate is altered by a new estimate:

$$T2 = T2 + (T_2/CP_2)\ .123406\ DSR \qquad \text{(Eq. D3)}$$

where: DSR = SR2 - SR1
CP2 was obtained by the entry into ANAPRP

This process is repeated until the entropy check is satisfied. Then all thermal and transport properties obtained from the last pass through (ANAPRP) the Mollier subroutine are the aft body, external to boundary layer properties.

APPENDIX E

Definition of Symbols in Program Output

A1W =	external to boundary layer speed of sound on fore wedge (ft/sec).
A2W =	external to boundary layer speed of sound on aft wedge (ft/sec).
AC1 =	external to boundary layer speed of sound on fore cone (ft/sec).
AC2 =	external to boundary layer speed of sound on aft cone (asymptotic pressure) (ft/sec).
AINFY =	free stream speed of sound (ft/sec).
ALPHA =	vehicle angle of attack (deg).
A2C =	external to boundary layer speed of sound on aft cone (P.M. pressure) (ft/sec).
CFLAM1 =	laminar friction coefficient on the forebody (-).
CFLAM2 =	laminar friction coefficient on the aftbody (-).
CFT1 =	turbulent friction coefficient on the forebody (-).
CFT2 =	turbulent friction coefficient on the aftbody (-).
CPCVC1 =	external to boundary layer specific heat ratio on fore cone (-).
CPCVC2 =	external to boundary layer specific heat ratio on aft cone (asymptotic pressure) (-).
CPC1 =	external to boundary layer specific heat on fore cone (BTU/lbm $^{\circ}$K).
CPC2 =	external to boundary layer specific heat on aft cone (asymptotic pressure) (BTU/lbm $^{\circ}$K).
CPCV2C =	external to boundary layer specific heat ratio on aft cone (P.M. pressure) (-).
CPCV1W =	external to boundary layer specific heat ratio on fore wedge (-).
CPCV2W =	external to boundary layer specific heat ratio on aft wedge (-).
CP1W =	external to boundary layer specific heat on fore wedge (BTU/lbm $^{\circ}$K).
CP2W =	external to boundary layer specific heat on aft wedge (BTU/lbm $^{\circ}$K).
CPINF =	free stream specific heat (BTU/lbm $^{\circ}$K).
CPREF1 =	specific heat at reference enthalpy and local pressure in the forebody boundary layer (BTU/lbm $^{\circ}$K).
CPREF2 =	specific heat at reference enthalpy and local pressure in the aftbody boundary layer (BTU/lbm $^{\circ}$K).
CPW1 =	specific heat at forebody wall temperature and local pressure (BTU/lbm $^{\circ}$K).
CPW2 =	specific heat at aftbody wall temperature and local pressure (BTU/lbm $^{\circ}$K).
DELS1 =	distance between stations to be calculated on forebody (ft).
DELS2 =	distance between stations to be calculated on aftbody (ft).
GAMINF =	free stream specific heat ratio (-).
HC1 =	external to boundary layer enthalpy on fore cone (BTU/lbm).
HC2 =	external to boundary layer enthalpy on aft cone (BTU/lbm).
H2C =	external to boundary layer enthalpy on aft cone using P.M. expansion pressure (BTU/lbm).

HINFY =	free stream enthalpy (BTU/lbm).
HREF1 =	boundary layer reference enthalpy on forebody (BTU/lbm).
HREF2 =	boundary layer reference enthalpy on aftbody (BTU/lbm).
HTOT =	free stream total enthalpy (BTU/lbm).
HW1 =	enthalpy at forebody wall temperature and local pressure (BTU/lbm).
HW2 =	enthalpy at aftbody wall temperature and local pressure (BTU/lbm).
H1W =	external to boundary layer enthalpy on fore wedge (BTU/lbm).
H2W =	external to boundary layer enthalpy on aft wedge (BTU/lbm).
PC1 =	Pressure on fore cone (asymptotic) (atm.).
PC2 =	Pressure on aft cone (asymptotic) (atm.).
P2C =	Pressure on aft cone from P.M. expansion (atm.).
PINFY =	free stream pressure (atm.).
PLOCC2 =	local pressure on aft cone as derived by shock-expansion method (atm.).
PRC1 =	external to boundary layer Prandtl number on fore cone (-).
PRC2 =	external to boundary layer Prandtl number on aft cone (-).
PR2C =	external to boundary layer Prandtl number on aft cone from Prandtl-Meyer expansion (-).
PR1W =	external to boundary layer Prandtl number on fore wedge (-).
PR2W =	external to boundary layer Prandtl number on aft wedge (-).
PRINF =	free stream Prandtl number (-).
PRREF1 =	Prandtl number at reference enthalpy and local pressure on fore body (-).
PRREF2 =	Prandtl number at reference enthalpy and local pressure on aft body (-).
PSIL1 =	recovery enthalpy minus wall enthalpy on fore body in laminar flow (BTU/lbm).
PSIL2 =	recovery enthalpy minus wall enthalpy on aft body in laminar flow (BTU/lbm).
PSIT1 =	recovery enthalpy minus wall enthalpy on fore body in turbulent flow (BTU/lbm).
PSIT2 =	recovery enthalpy minus wall enthalpy on aft body in turbulent flow (BTU/lbm).
P1W =	Pressure on fore wedge (atm.).
P2W =	Pressure on aft wedge (atm.).
QRTL1 =	ratio of laminar heat rate at angle of attack to that at zero angle of attack on fore body (-).
QRTL2 =	same as QRTL1 but considers aft body (-).
QRTT1 =	ratio of turbulent heat rate at angle of attack to that at zero angle of attack on fore cone (-).
QRTT2 =	Same as QRTT1 but considers aft body (-).
QSLAM1 =	local heat transfer rate from a laminar boundary layer on the fore body (BTU/ft^2 sec).
QSLAM2 =	local heat transfer rate from a laminar boundary layer on the aft body (BTU/ft^2 sec).
QST1 =	local heat transfer rate from a turbulent boundary layer on the fore body (BTU/ft^2 sec).
QST2 =	local heat transfer rate from a turbulent boundary layer on the aft body (BTU/ft^2 sec).
REY1 =	local Reynolds number on fore body (-).
REY2 =	local Reynolds number on aft body (-)
RHOC1 =	external to boundary layer density on fore cone (slugs/ft^3).

RHOC2 = external to boundary layer density on aft cone (asymptotic pressure)(slugs/ft^3).

RHO2C = external to boundary layer density on aft cone (PM pressure) (slugs/ft^3).

RHOINF = free stream density (slugs/ft^3).

RHOR1 = density at reference enthalpy and local pressure in fore body boundary layer (slugs/ft^3).

RHOR2 = density at reference enthalpy and local pressure in aft body boundary layer (slugs/ft^3).

RHOW1 = density at fore body wall temperature and local pressure (slugs/ft^3).

RHOW2 = density at aft body wall temperature and local pressure (using either P.M. or asymptotic, as user selects), (slugs/ft^3).

RHO1W = external to boundary layer density on fore wedge (slugs/ft^3).

RHO2W = external to boundary layer density on aft wedge (slugs/ft^3).

S = surface coordinate distance to point being investigated (ft).

SO = surface coordinate distance from nose stagnation point to fore body - nose shoulder (ft).

S1 = surface coordinate distance from nose apex to fore body - aft body shoulder (ft).

S2 = surface coordinate distance from nose apex to aft end of aft body (ft).

S2MINL = effective boundary layer build-up distance at start of downstream body such that momentum defect is constant across the expansion for laminar flow (ft).

S2MINT = same as S2MINL but for a turbulent boundary layer (ft).

SRINF = nondimensionalized free stream entropy (S/R) (-).

SRC1 = nondimensionalized external to boundary layer entropy (S/R) on fore cone (-).

SRC2 = nondimensionalized external to boundary layer entropy (S/R) on aft cone using asymptotic aft cone pressure (-).

SR2C = same as SRC2 but using P.M. pressure on aft cone (-).

SR1W = same as SRC1 but considers fore body as wedge (-).

SR2W = same as SR1W but the aftbody is a wedge (-).

TAUL1 = local shear stress on fore body for laminar flow (lbf/ft^2).

TAUL2 = local shear stress on aft body for laminar flow (lbf/ft^2).

TAUT1 = local shear stress on fore body for turbulent flow (lbf/ft^2).

TAUT2 = local shear stress on aft body for turbulent flow (lbf/ft^2).

TC1 = external to boundary layer temperature on fore cone (^oK).

TC2 = external to boundary layer temperature on aft cone (asymptotic pressure) (^oK).

TETC1 = fore body angle plus angle of attack (deg.).

TETC2 = aft body half angle plus angle of attack (deg.).

TETSC1 = fore cone shock angle (deg.).

TETSC2 = aft cone shock angle (deg.).

TETS1W = fore wedge shock angle (deg.).

TETS2W = aft wedge shock angle (deg.).

TINFY = free stream temperature (^oK).

TKC1 = external to boundary layer thermal conductivity on fore cone (BTU/ft sec ^oK).

TKC2 = external to boundary layer thermal conductivity on aft cone (asymptotic pressure) (BTU/ft sec ^oK).

TK2C = external to boundary layer thermal conductivity on aft cone (P.M. pressure) (BTU/ft sec ^{o}K).

TKREF1 = thermal conductivity at reference enthalpy and local pressure in fore body boundary layer (BTU/ft sec ^{o}K).

TKREF2 = same as TKREF1 but for aft body (BTU/ft sec ^{o}K).

TK1W = external to boundary layer thermal conductivity on fore wedge (BTU/ft sec ^{o}K).

TK2W = external to boundary layer thermal conductivity on aft wedge (BTU/ft sec ^{o}K).

TREF1 = temperature at reference enthalpy and local pressure in the fore body boundary layer (^{o}K).

TREF2 = same as TREF1 but for aft body (^{o}K).

T2C = external to boundary layer temperature on aft cone (P.M. pressure) (^{o}K).

T1W = external to boundary layer temperature on fore wedge (^{o}K).

T2W = same as T1W but for aft wedge (^{o}K).

VC1 = external to boundary layer velocity on fore cone (ft/sec).

VC2 = external to boundary layer velocity on aft cone (asymptotic pressure)(ft/sec.).

V2C = external to boundary layer velocity on aft cone (P.M. pressure) (ft/sec.).

VISCC1 = external to boundary layer dynamic viscosity on fore cone (lbf sec/ft^2).

VISCC2 = external to boundary layer dynamic viscosity on aft cone (asymptotic pressure) lbf sec/ft^2).

VISC2C = external to boundary layer dynamic viscosity on aft cone (P.M. pressure) (lbf sec/ft^2).

VISCR1 = dynamic viscosity at reference enthalpy and local pressure in fore body boundary layer (lbf sec/ft^2).

VISCR2 = same as VISCR1 but for aft body (lbf sec/ft^2).

VSCINF = free stream dynamic viscosity (lbf sec/ft^2).

VISC1W = external to boundary layer dynamic viscosity on fore wedge (lbf sec/ft^2).

VISC2W = same as VISC1W but on aft wedge (lbf sec/ft^2).

V1W = external to boundary layer velocity on fore wedge (ft/sec).

V2W = same as V1W but on aft wedge (ft/sec).

X1 = coordinate distance to point being investigated along a free stream velocity vector through the nose stagnation point for fore body (ft).

X2 = same as X1 but for points on aft body (ft).

XMC1 = external to boundary layer Mach number on fore cone (-).

XMC2 = same as XMC1 but on aft body and using asymptotic pressure (-).

XM2C = same as XMC2 but for P.M. pressure (-).

XM1W = external to boundary layer Mach number on fore wedge (-).

XM2W = same as XM1W but for aft wedge (-).

XMINF = free stream Mach number (-).

ZC1 = external to boundary layer compressibility factor on fore cone (-).

ZC2 = external to boundary layer compressibility factor on aft cone (asymptotic pressure) (-).

Z2C = same as ZC2 but using P.M. pressure (-).

ZINF = free stream compressibility factor (-).

Z1W = external to boundary layer compressibility factor on fore wedge (-).

Z2W = same as Z1W but for aft wedge (-).

IBM FORTRAN Coding Form

PROGRAM NZLDW005 - INPUT SUMMARY

PROGRAMMER | DATE | PUNCHING INSTRUCTIONS | GRAPHIC | PUNCH | PAGE OF | CARD ELECTRO NUMBER*

STATEMENT NUMBER	FORTRAN STATEMENT				IDENTIFICATION SEQUENCE
N NTUN	TITLE				CARD TYPE 1
ALT (FT)	VINFY (FT/SEC)	DEL (-)	ALPHA (DEG)		CARD TYPE 2
TETC1 (DEG)	TETC2 (DEG)	S 0 (FT)	S 1 (FT)		
S 2 (FT)	DELS 1 (FT)	DELTS2 (FT)	TW1 (^oK)		
TW 2 (^oK)	LLL JJJ LDW				
PINFY (ATM)	TINFY (^oK)				CARD TYPE 3

ITEM	FORMAT
N = 0 for wedge flow; = 1 for cone flow	Fixed point (use col. 2 only)
NTUN = 0 for 1959 ARDC atmos.; = 1 for tunnel cond.'s	Fixed point (use col. 4 only)
Title = Arbitrary alpha-numeric characters	Alpha-Numeric; use col.s 7-72
ALT = vehicle altitude (ft)	Floating point col.s 1-15
Vinfy = vehicle free stream velocity (ft/sec)	Floating point col.s 16-30
Del = step function for shock routine	Floating point col.s 31-45
Alpha = vehicle angle of attack (deg)	Floating point col.s 46-60
TETC1 = forebody half-angle + alpha (deg)	Floating point col.s 1-15
TETC2 = Aftbody half-angle + alpha (deg)	Floating point col.s 16-30
SØ = Surf. coordinate dist. from nose apex to forebody - nose shoulder (ft)	Floating point col.s 31-45
S1 = Surf. coordinate dist. from nose apex to forebody - aftbody shoulder (ft)	Floating point col.s 45-60
S2 = Surf. coordinate dist. from nose apex to aft end of aftbody (ft)	Floating point col.s 1-15
Delts1 = Step in S to station forebody (ft)	Floating point col.s 16-30
Delts2 = Step in S to station aftbody (ft)	Floating point col.s 31-45
TW1 = Surf. Temperature on forebody (^oK)	Floating point col.s 46-60
TW2 = Surf. Temperature on aftbody (^oK)	Floating point col.s 1-15
LLL = 0 (no dump); 1 (dump) iterations in subroutine ITER	Fixed point (right justify in columns 16-17)
JJJ = 0 (P-M press. on aftbody) = 1 (asymptotic press. on aftbody	Fixed point (right justify in columns 18-19)
Pinfy = ambient press. (atm) } enter only if	Floating point col.s 1-15
Tinfy = ambient Temp. (^oK) } NTUN = 1	Floating point col.s 16-30
LDW = 0 (program interupts allowed) = 1 to suppress interupts (normally left blank)	

NOTES:

a. Only card type 3 (card no. 6) is optional (use only when NTUN = 1). All other card (1-05) must always be entered.

b. Problems set up as though for individual run may be stacked in any number of problems.

c. SØ > 0

d. TETC1 > TETC2

*A standard card form, IBM electro 888157, is available for punching statements from this form

FIGURE 2
INPUT SUMMARY